Convergence issues in ChPT: a lattice perspective

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Overview (1): where LQCD stands

LAT'01 (Berlin)

$N_f = 2$ in sea (in functional determinant)

Algorithms scale badly with m_q and a: In those days $500\,\mathrm{MeV} \leq M_\pi$ and $a \simeq 0.12\,\mathrm{fm}$, i.e. too narrow database to extract physics.

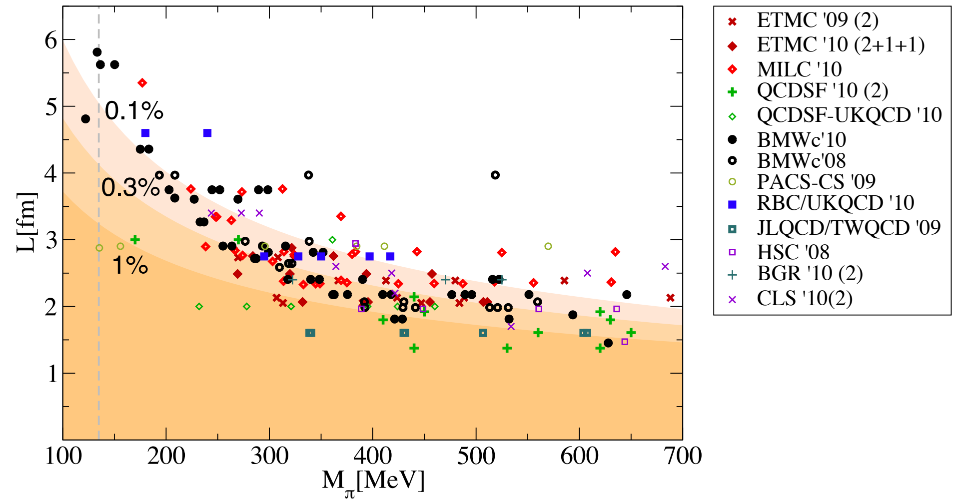
LAT'11 (Squaw Valley)

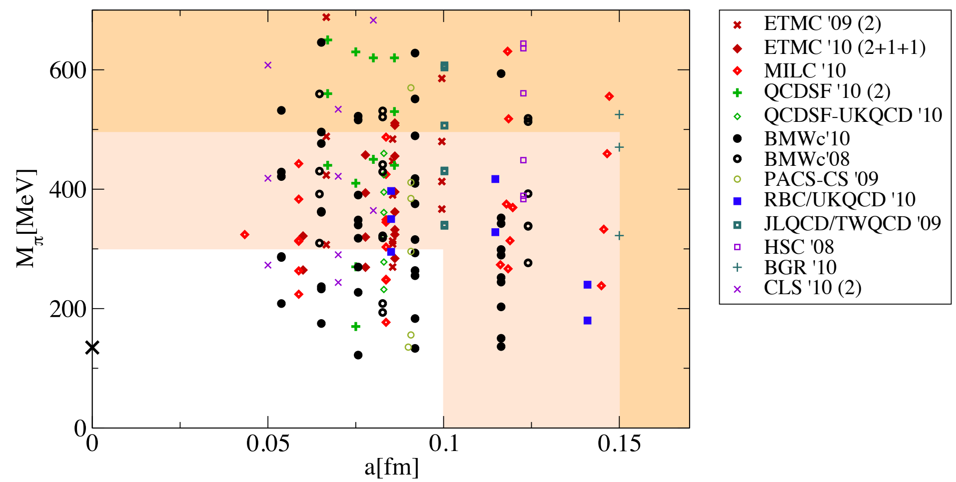
$N_f = 2+1$ in sea (relevant dofs included)

Algorithmic bottlenecks mostly overcome: Nowadays $120\,\mathrm{MeV} \leq M_\pi \leq 400\,\mathrm{MeV}$ for interpolation to M_π^{phys}, and $0.05\,\mathrm{fm} \leq a \leq 0.12\,\mathrm{fm}$ for controlled continuum limit, and $2\,\mathrm{fm} \leq L \leq 6\,\mathrm{fm}$ to limit finite-size effects.

$\rightarrow$ LQCD gives results from first principles.

Fodor Hoelbling, Review'12





Nowadays question is: Who is useful to whom in LQCD $\leftrightarrow$ ChPT ?

Overview (2): Principles of mesonic ChPT

ChPT provides a rigorous framework to compute Green's functions of QCD, based on (i) symmetry, (ii) analyticity, (iii) [perturbative] unitarity.

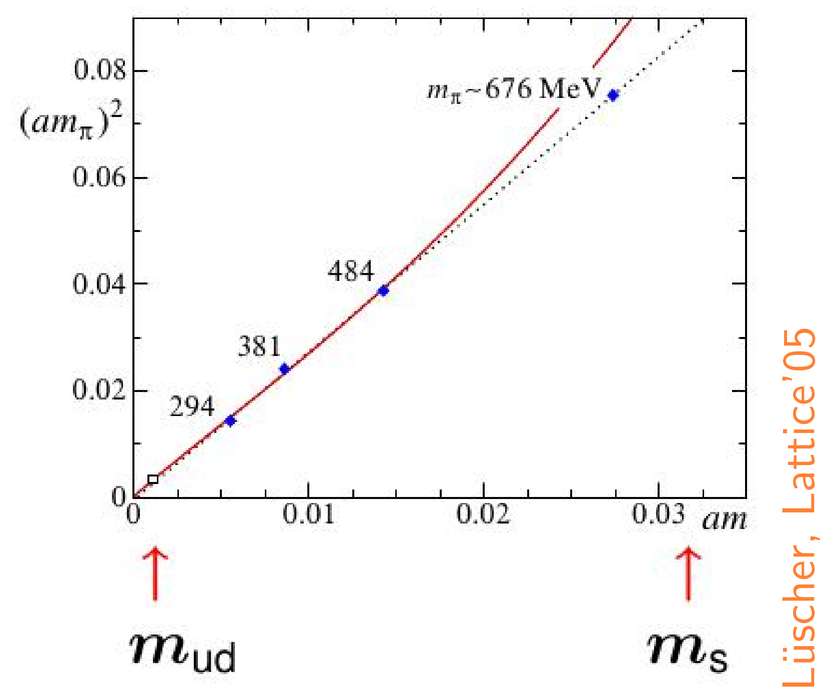
Low-energy constants (LECs) govern momentum and quark-mass dependence:
$\longrightarrow$ p-dependence mapped out in experiment; obtain respective LECs from experiment.
$\longrightarrow$ m_q-dependence in nature only in discrete steps; get respective LECs from lattice.

Correct counting rule is $p^2 \sim m$, provided $B = \Sigma/F^2$ is dominant symmetry breaking.
$\longrightarrow$ lattice can check by measuring B (and F and NLO/NNLO low-energy constants).

Main finding of all lattice studies:

M_π^2 in dynamical (as well as quenched) studies in excellent approximation *linear* in bare or renormalized quark mass m_q.

Slope yields LO constant B (or B_0) in same scheme (cut-off, $\overline{\mathrm{MS}}$, etc.) as m_q. Tiny deviation from linearity yields $\bar{\ell}_3$.



Overview (3): Versions of mesonic ChPT

- Chiral SU(2) Lagrangian

  LO: 2 LECs [F and $B = \Sigma/F^2$, specific to $m_{ud} \to 0$, often denoted F_2, B_2]
  NLO: 7 LECs [$\ell^{\text{ren}}_{1,...,7}(\mu)$ or $\bar{\ell}_{1,...,7}$]
  NNLO: plethora of LECs
  Value $m_{ud}^{\text{phys}} \simeq 3.5\,\text{MeV}$ ($\overline{\text{MS}}$, 2 GeV) small enough for good convergence.
  Phenomenological LECs depend implicitly on m_s^{phys} and heavier flavors.

- Chiral SU(3) Lagrangian

  LO: 2 LECs [F_0 and $B_0 = \Sigma_0/F_0^2$, for $m_{ud}, m_s \to 0$, often denoted F_3, B_3]
  NLO: 10 LECs [$L^{\text{ren}}_{1,...,10}(\mu)$] plus 2 HECs [contact terms]
  NNLO: plethora of LECs
  Value $m_s^{\text{phys}} \simeq 95\,\text{MeV}$ ($\overline{\text{MS}}$, 2 GeV) at the edge of applicability window.
  Phenomenological LECs depend implicitly on m_c^{phys} and heavier flavors.

Note *hierarchy in precision* of LECs needed/determined: LO most accurate (e.g. permille level), NLO intermediate (e.g. percent level), NNLO order of magnitude.

Overview (4): Versions of LQCD simulations

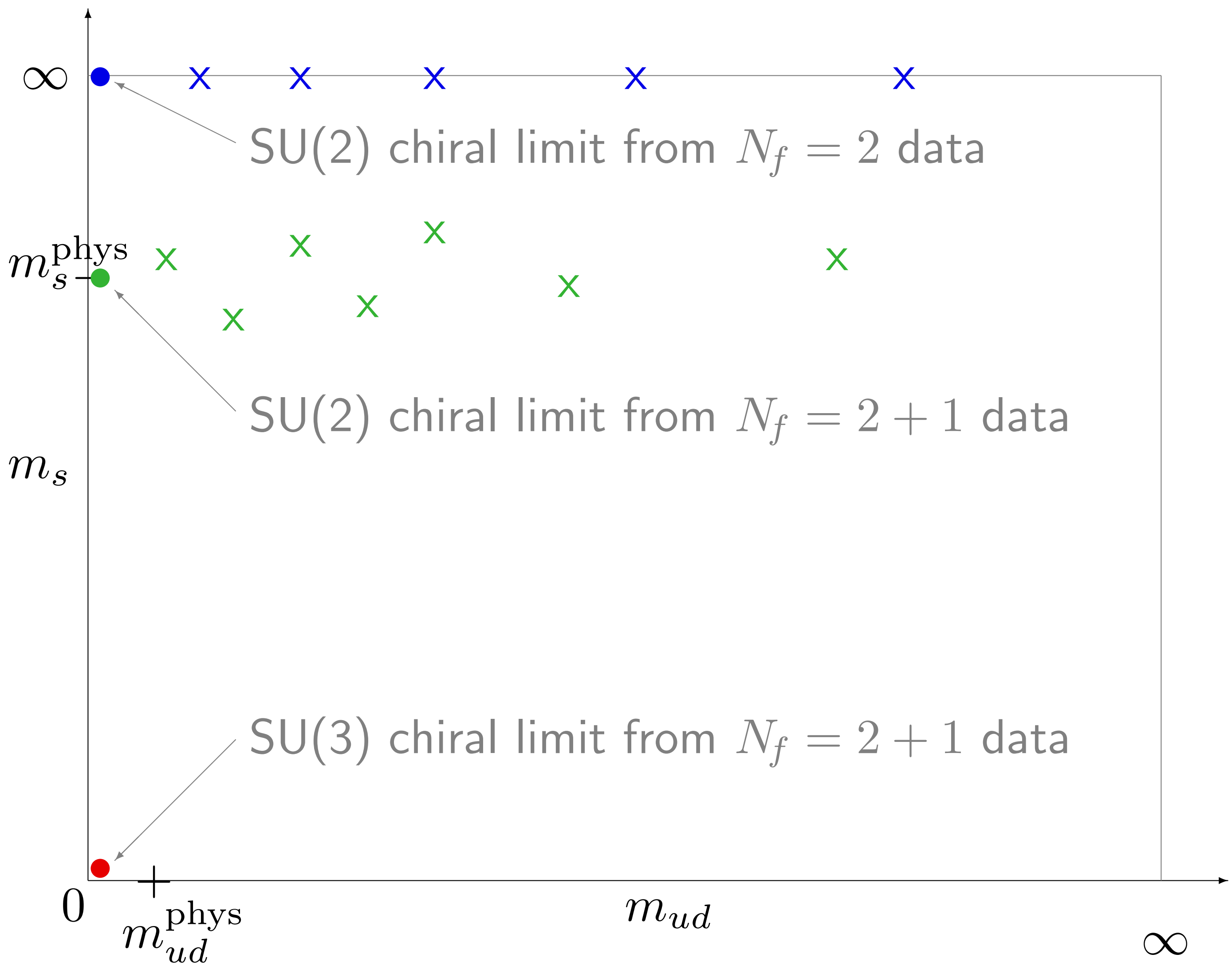
- $N_f = 2$ calculations (blue on cartoon):
  $m_{ud}^{\rm sea}$ in functional determinant, same $m_{ud}^{\rm val}$ in hadronic correlators.
  Flavors s, c, b only in correlators (quenched) or absent (unitary theory).
  $\longrightarrow$ analysis must use SU(2) ChPT; these l_i differ from those in phenomenology.

- $N_f = 2+1$ calculations (green on cartoon):
  $m_{ud}^{\rm sea}$, $m_s^{\rm sea}$ in functional determinant, same $m_{ud}^{\rm val}$, $m_s^{\rm val}$ in correlators.
  Flavors c, b only in correlators (quenched) or absent (unitary theory).
  $\longrightarrow$ analysis may use SU(2)-ChPT; these l_i coincide (almost) with phenomenology.
  $\longrightarrow$ analysis may use SU(3)-ChPT; these L_i coincide (almost) with phenomenology.

- $N_f = 2+1+1$ calculations:
  $m_{ud}^{\rm sea}$, $m_s^{\rm sea}$, $m_c^{\rm sea}$ in functional determinant, same $m_{ud}^{\rm val}$, $m_s^{\rm val}$, $m_c^{\rm val}$ in correlators.
  Unitary theory involves flavors up to c; both SU(2) and SU(3) framework legitimate.

- $N_f = 1+1+1+1$ plus (quenched or full) QED calculations:
  Same as in previous item, except that $m_u \neq m_d$ (both in sea and valence sector).
  Makes only sense if electromagnetic interactions included too [$\longrightarrow$ Antonin Portelli].

[Versions of LQCD simulations, continued]

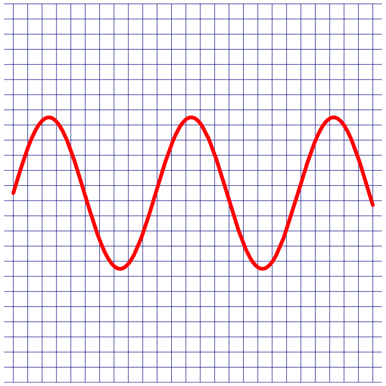


Attention: further convergence issues with $m^{\rm sea} \neq m^{\rm val}$ ("partial quenching").
Attention: further convergence issues with finite-a effects in ChPT framework.

Overview (5): ChPT in finite (euclidean) volume

New player: spatial/euclidean box-length L.
Minimum momentum (with periodic boundary conditions) is $p_{\mathrm{min}} = 2\pi/L$.
Example: $L = 2\,\mathrm{fm}$ means $p_{\mathrm{min}} \simeq 2\pi\,100\,\mathrm{MeV} \simeq 630\,\mathrm{MeV}$ [edge of applicab. of ChPT].

Finite volume effect on meson masses: $M_\pi(L) > M_\pi \equiv M_\pi(\infty)$.
Finite volume effect on decay constant: $F_\pi(L) < F_\pi \equiv F_\pi(\infty)$.



- p-regime: $1 \ll M_\pi L \ll 4\pi F_\pi L$, count $M_\pi^2 \sim p^2 \sim 1/L^2$.
  Box large both in absolute units and relative to $\xi_\pi \equiv \xi_\pi(\infty)$.
  Hierarchy of difficulties to access LECs at LO/NLO/NNLO.

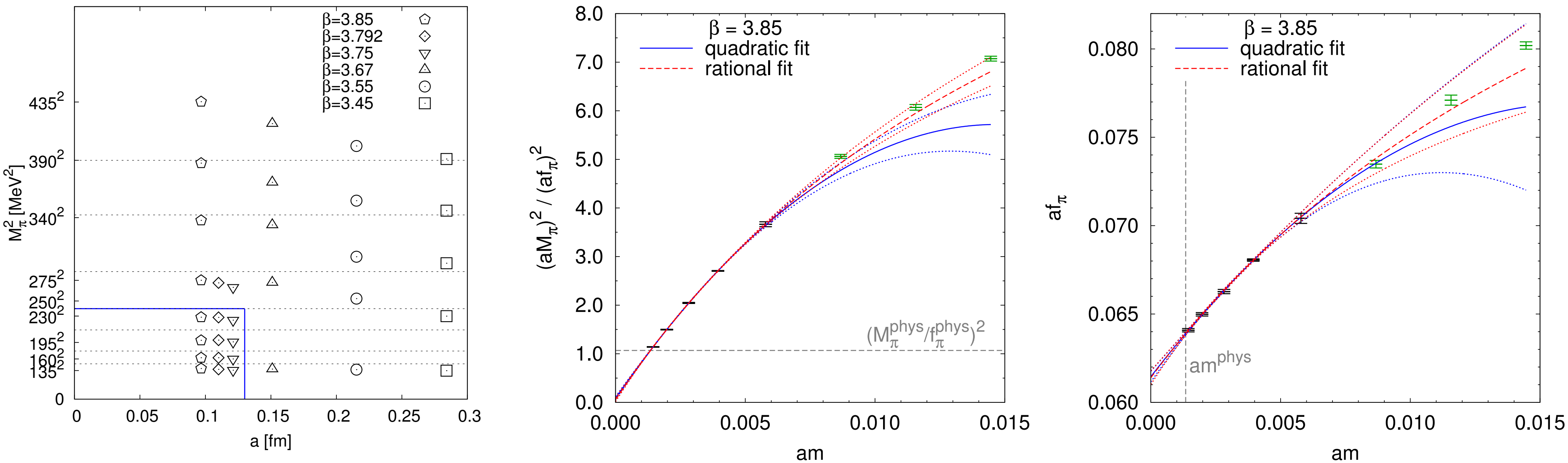
- ϵ-regime: $M_\pi L \ll 1 \ll 4\pi F_\pi L$, count $M_\pi \sim p^2 \sim 1/L^2 \sim \epsilon^2$.
  Box large in absolute units but small relative to $\xi_\pi \equiv \xi_\pi(\infty)$.
  Reordering gives preferred access to LO constants $B = \Sigma/F^2$ and F.

- δ-regime: $M_\pi L_s \ll 1 \ll M_\pi L_t \ll 4\pi F_\pi \{L_s, L_t\}$
  Ditto for spatial extent, but not for temporal extent.
  Physics of quantum mechanical rotator [Leutwyler, Hasenfratz].

Success with SU(2) ChPT (1): WBR calculation

Borsanyi, Durr, Scholz *et al.* [Wuppertal-Budapest-Regensburg], arXiv:1205.0788.

- **Simulation landscape and scale setting**



Series of staggered $N_f = 2+1$ simulations at various $a = a(\beta)$, $m_{ud}^{\rm sea} = m_{ud}^{\rm val}$ varies, $m_s^{\rm sea} = m_s^{\rm val}$ tuned to physical value.

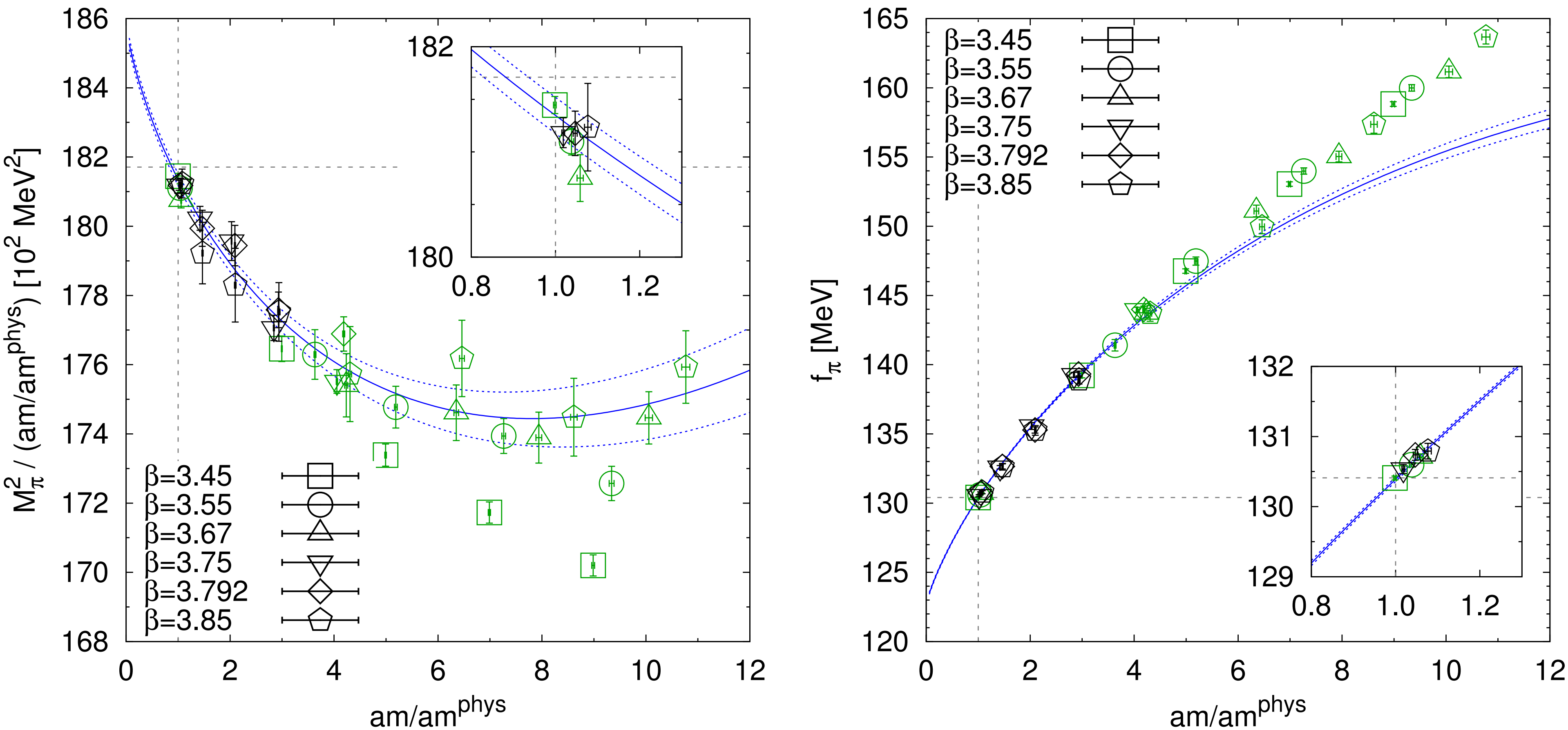
For each β: Interpolate M_π^2/f_π^2 to the point where it assumes its physical value 1.06846, read off $(am)^{\rm phys}$.

For each β: Determine af_π for that $(am)^{\rm phys}$ and thus a via $f_\pi^{\rm PDG}$

• **Joint NLO SU(2) chiral fit**

$$M_\pi^2 = (aM_\pi)^2/a^2 = \chi\left[1+\frac{\chi}{16\pi^2 f^2}\log\frac{\chi}{\Lambda_3^2}\right]$$

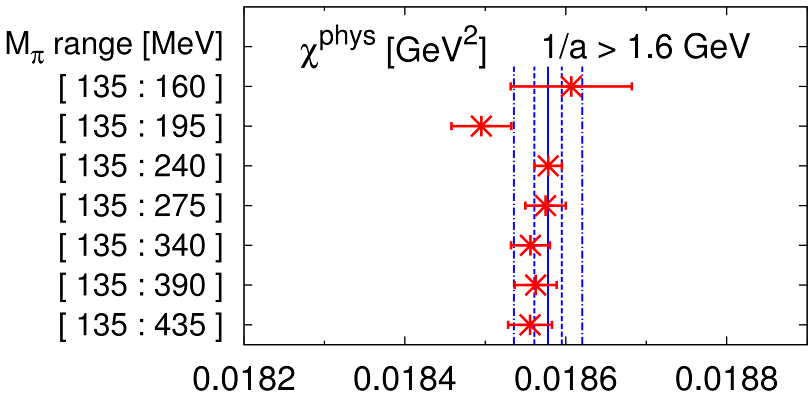
$$f_\pi = (af_\pi)/a = f\left[1-\frac{\chi}{8\pi^2 f^2}\log\frac{\chi}{\Lambda_4^2}\right],\quad \chi = 2B\,m = (2Bm^{\rm phys})\,\frac{am}{am^{\rm phys}}$$

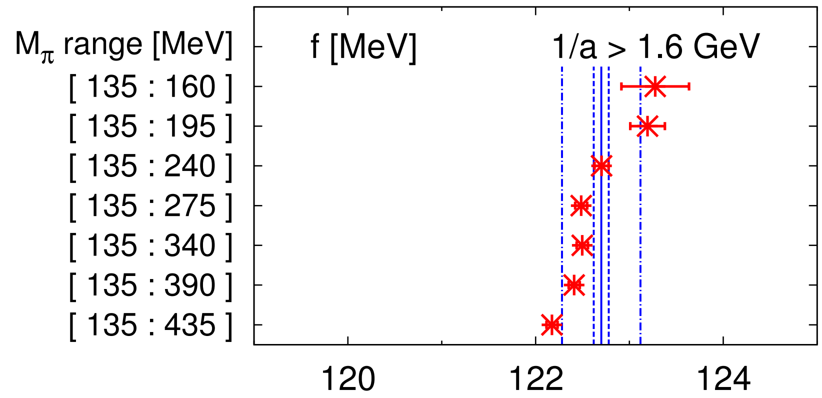


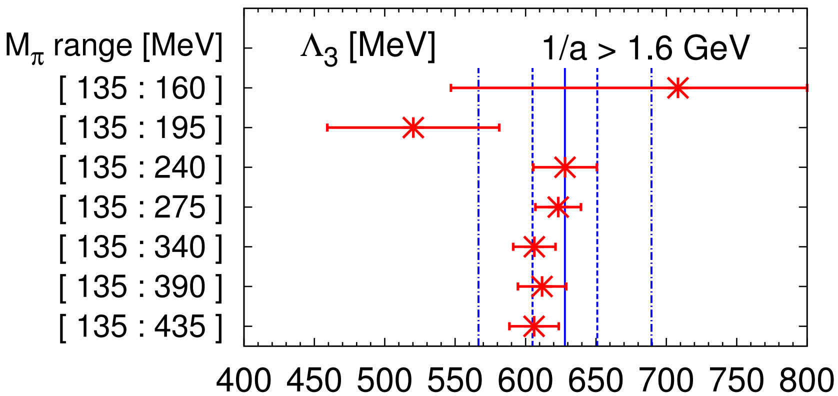
Joint fit to $F_\pi = F_\pi(m)$ and $M_\pi^2 = M_\pi^2(m)$ yields B, F and $\bar{\ell}_3$, $\bar{\ell}_4$ or Λ_3, Λ_4.

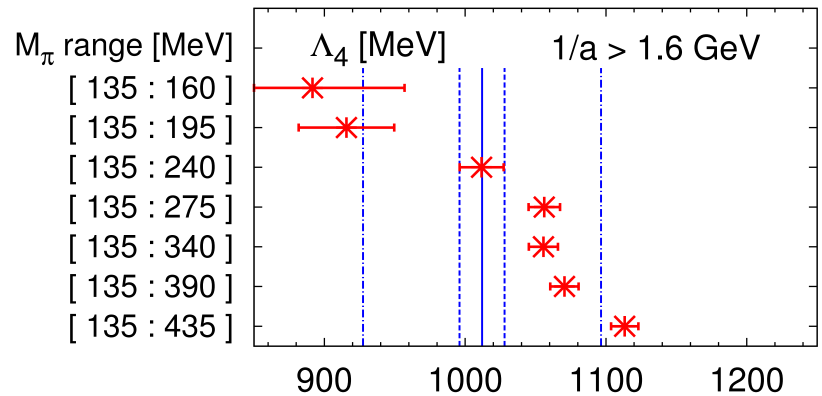
Only acceptable after restriction to $1.6\,\mathrm{GeV} < a^{-1}$ and $M_\pi \leq 240\,\mathrm{MeV}$ (black data).

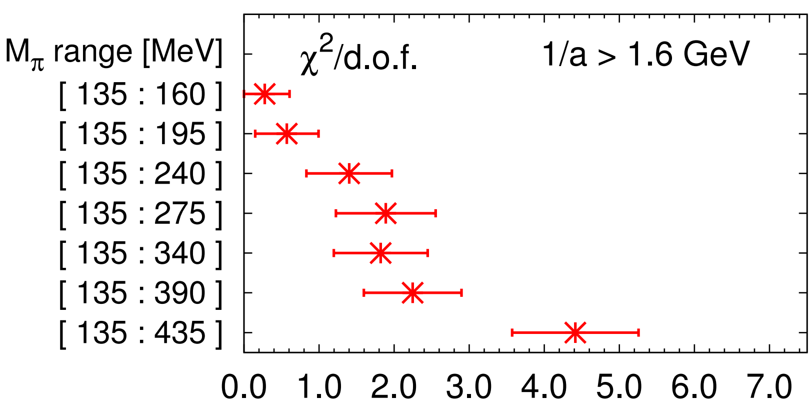
• Dependence of LO/NLO parameters on chiral range









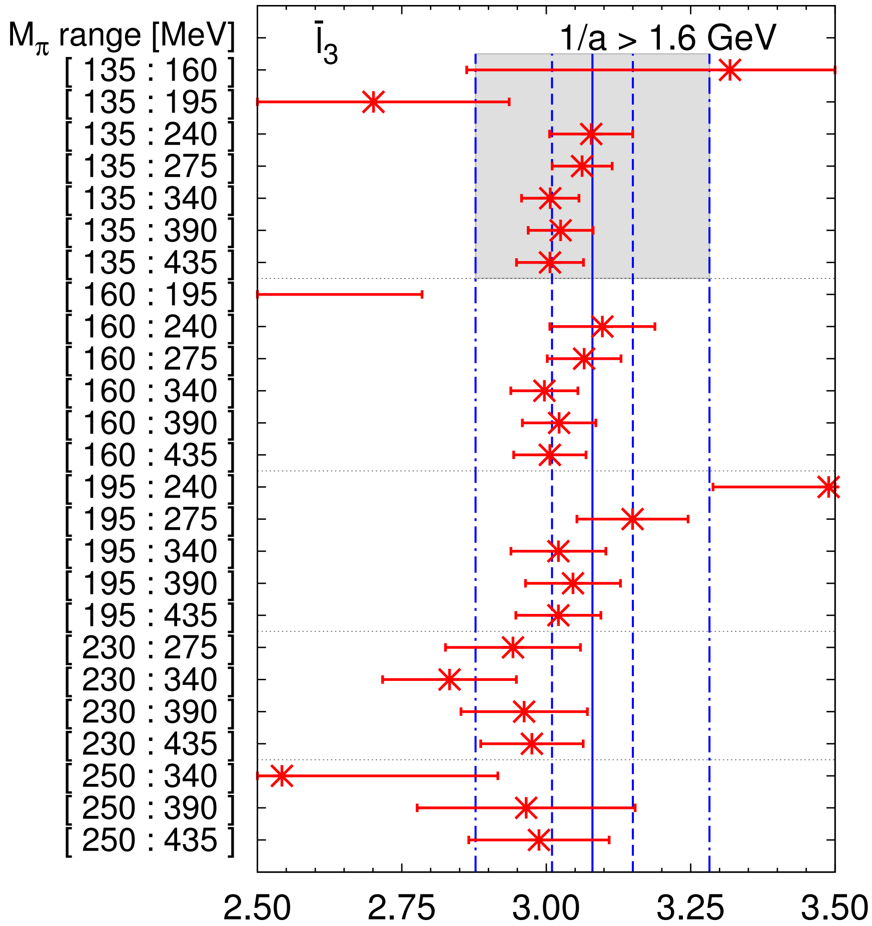


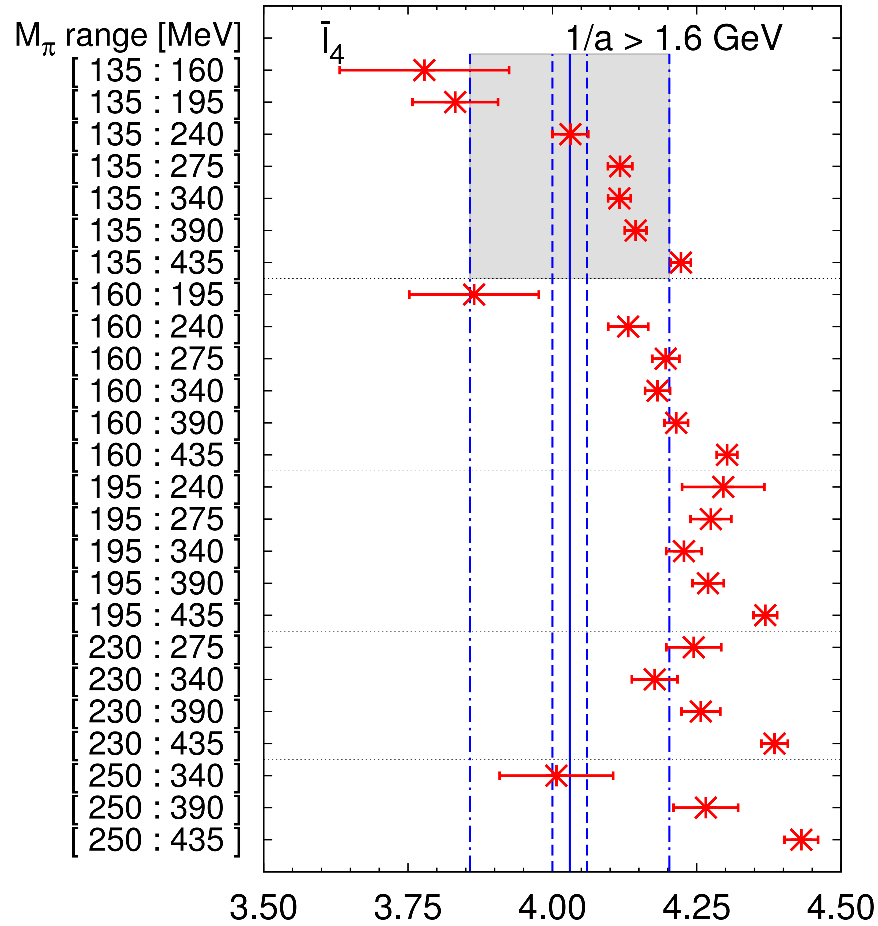
Central value and statistical error from [135:240]MeV fit (with $a^{-1} > 1.6\,\mathrm{GeV}$).

Systematic error from width of distribution.

χ^2/dof consistent with 1 within stat. error.

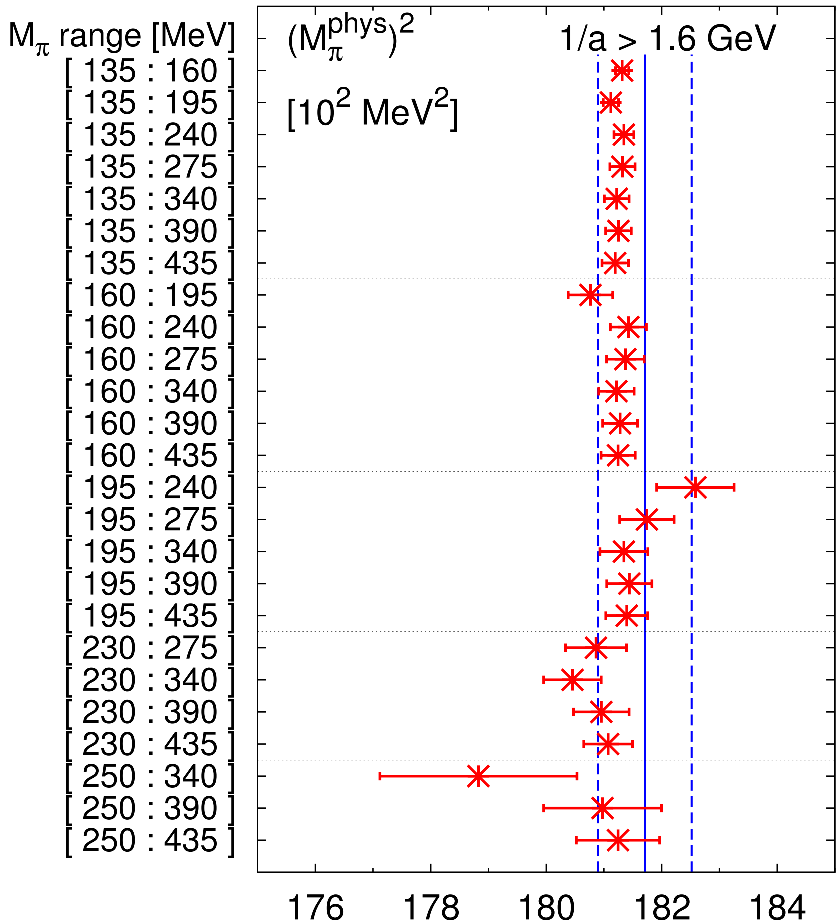
• Dependence of NLO parameters on (extended) chiral range

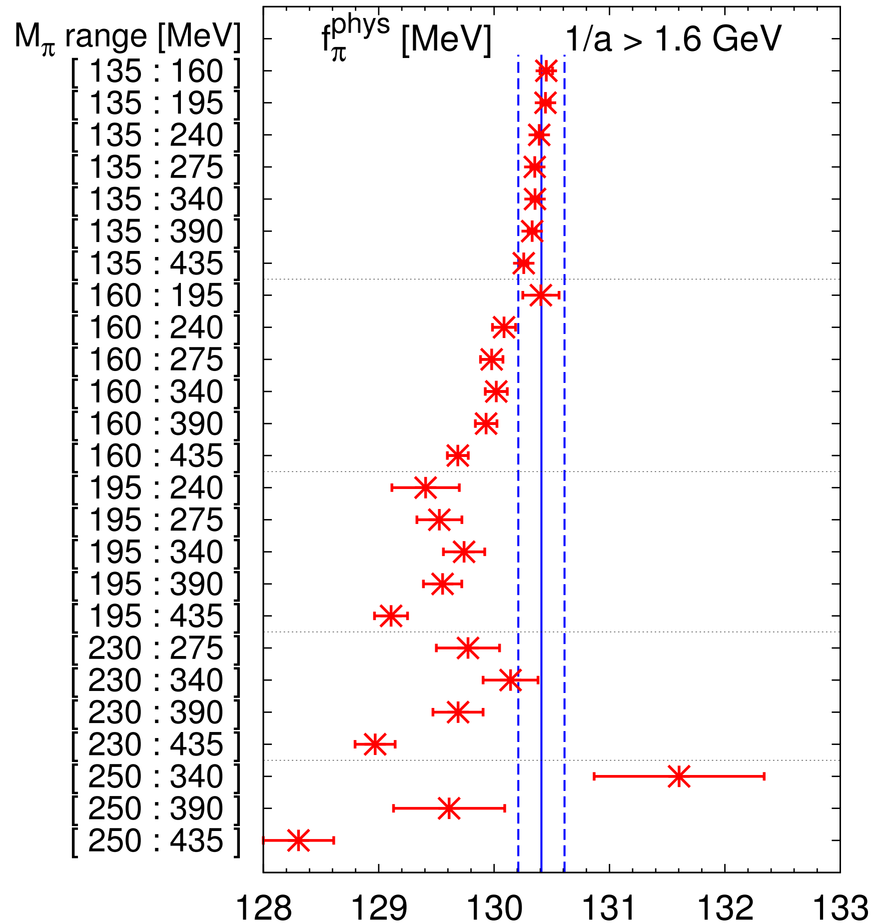




Deliberately leaving out near-physical mass points barely affects results for $\bar{\ell}_3$ (left), but increases instability for $\bar{\ell}_4$ (right).

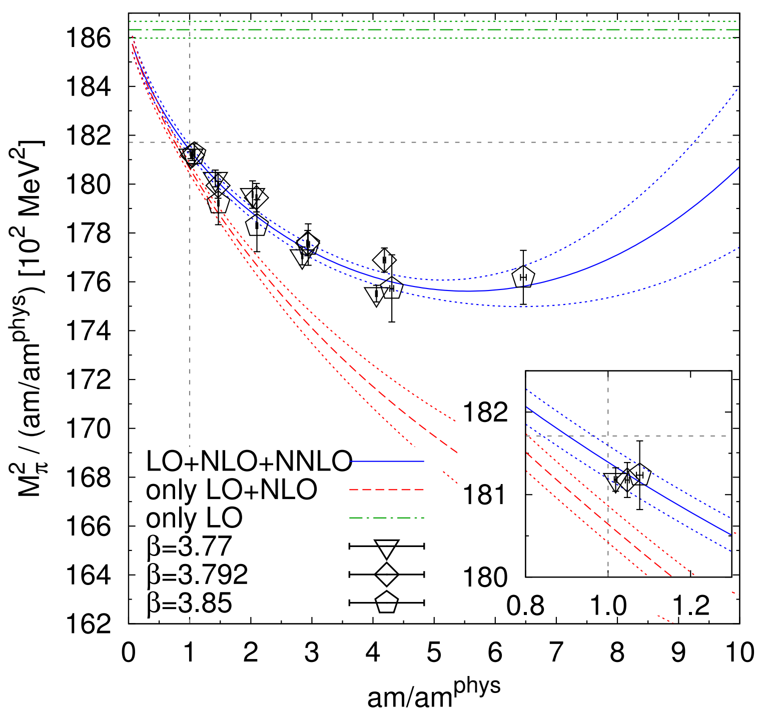
• Dependence of M_π^{phys} and F_π^{phys} on (extended) chiral range

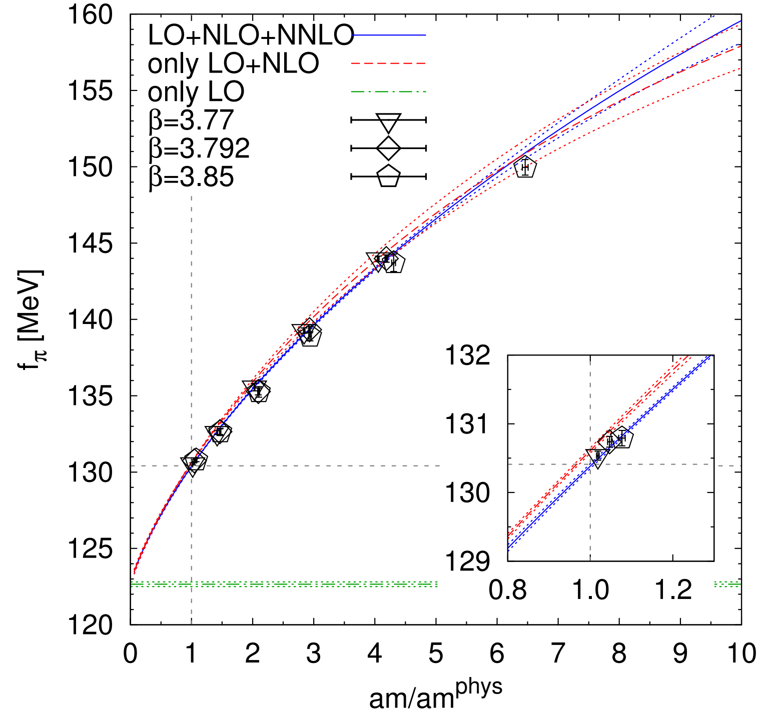


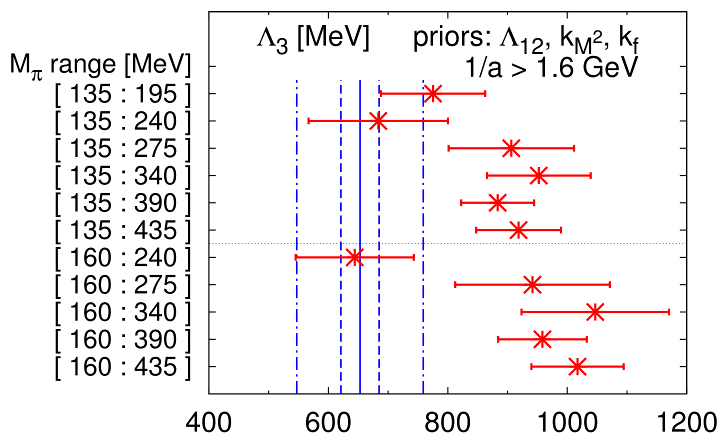


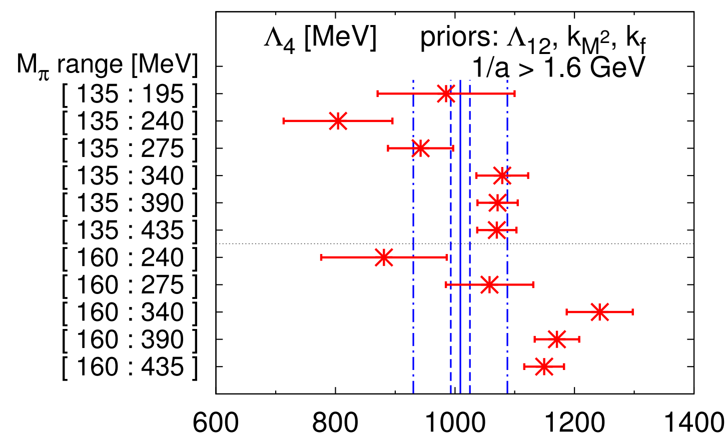
Deliberately leaving out near-physical mass points lets chiral NLO fits still estimate M_π^{phys} correctly (left), but leads to an underestimate of f_π^{phys} (right).

• Exploratory NNLO fits with NNLO-priors





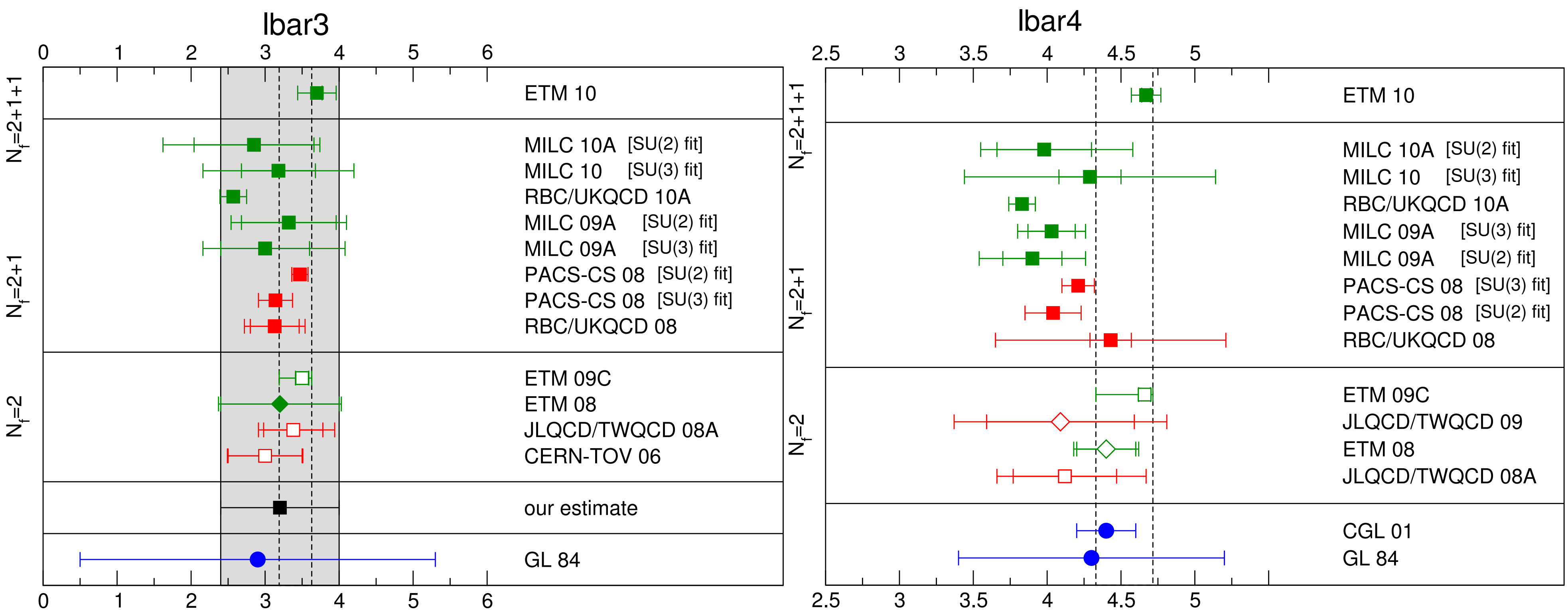




Split-up of LO+NLO+NNLO fit (priors for NNLO part) suggests good convergence at physical mass point and yields $\Lambda_{3,4}$ consistent with those from LO+NLO fit.

Success with SU(2) ChPT (2): FLAG summary

Colangelo *et al.* [FLAG consortium], Eur.Phys.J. C71 (2011) 1695 [arXiv:1011.4408]

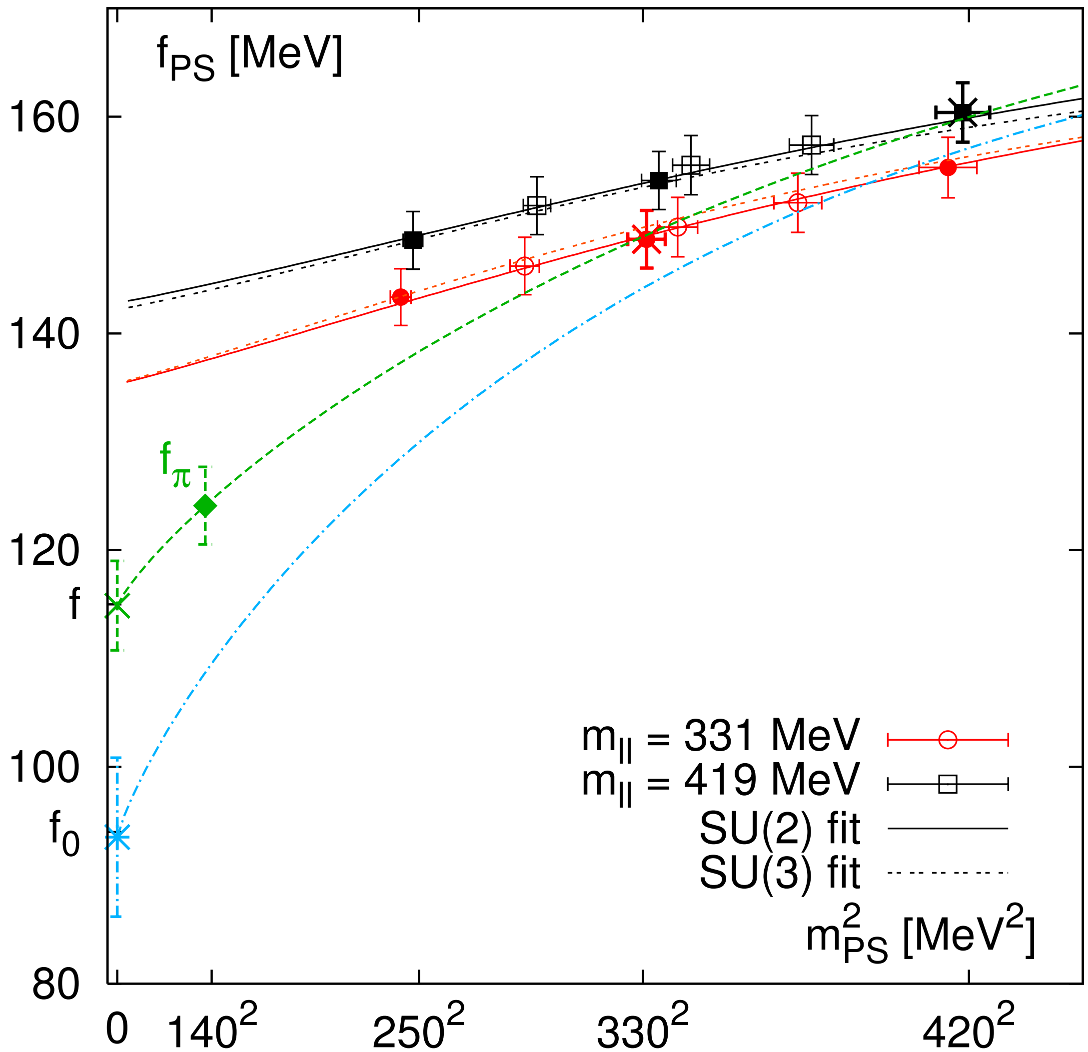


$\longrightarrow$ Reasonable consistency among all lattice determinations [sc. with $N_f = 2$, $N_f = 2{+}1$, $N_f = 2{+}1{+}1$] for $\bar{\ell}_3$ and (a bit less so) for $\bar{\ell}_4$. Ditto at LO.

$\longrightarrow$ Value of $\bar{\ell}_3$ is bang-on original GL'84 estimate, but with smaller error.

Issues with SU(3) ChPT (1): RBC/UKQCD calculation

Allton *et al.* [RBC/UKQCD], Phys.Rev. D78 (2008) 114509 [arXiv:0804.0473]



SU(2) fit (green): f_π and f considerably lower than data.

SU(3) fit (bluish): f_0 significantly lower than data.

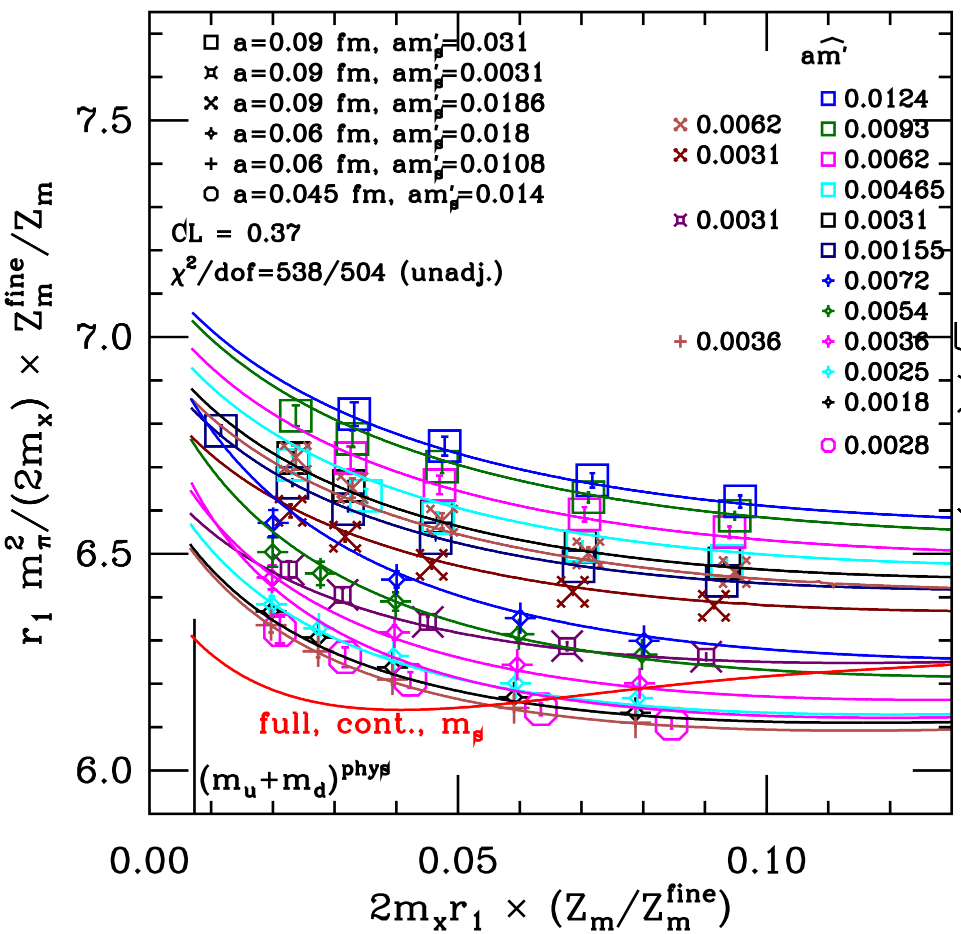
Fit seems to underestimate f and (even more so) f_0.

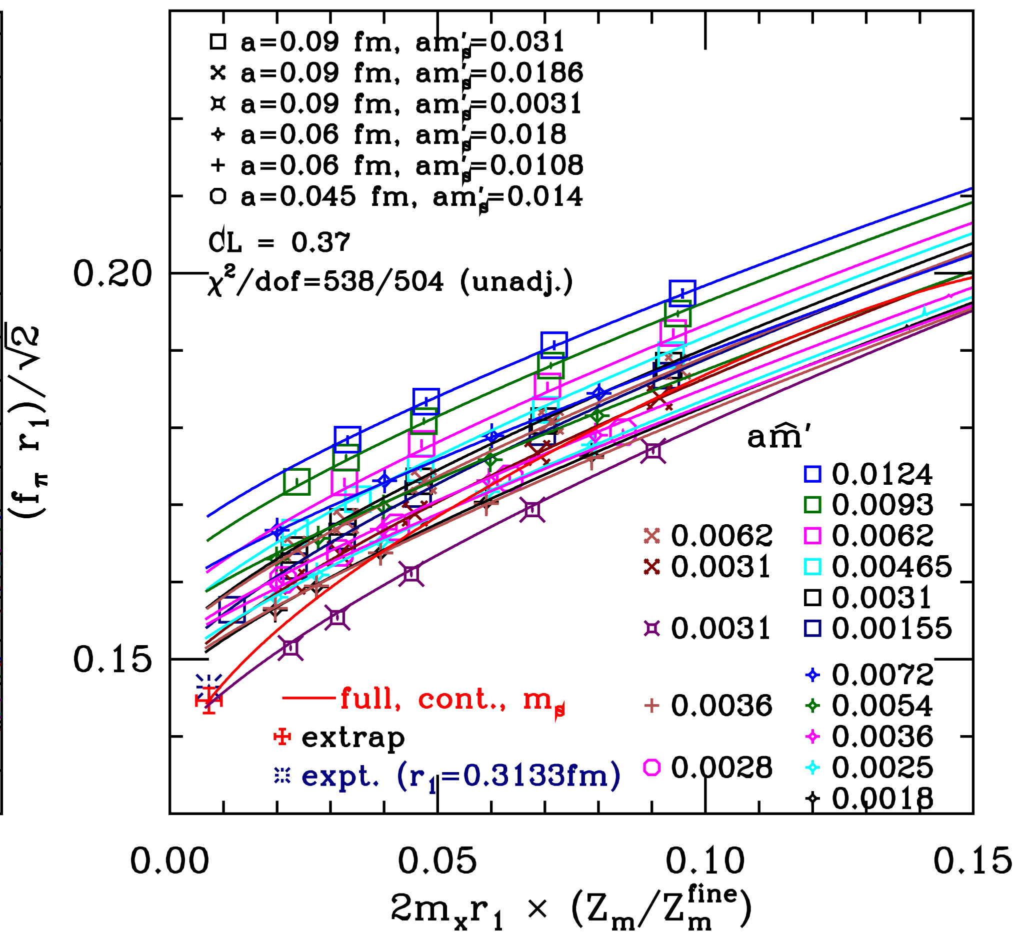
Authors conclude:
SU(3) ChPT converges poorly.

Delicate issues: (i) size of m_s, (ii) nun-unitarity due to $m^{\rm sea} \neq m^{\rm val}$.

Issues with SU(3) ChPT (2): MILC calculation

Bernard *et al.* [MILC], PoS Lattice2010 (2010) 074 [arXiv:1012.0868]

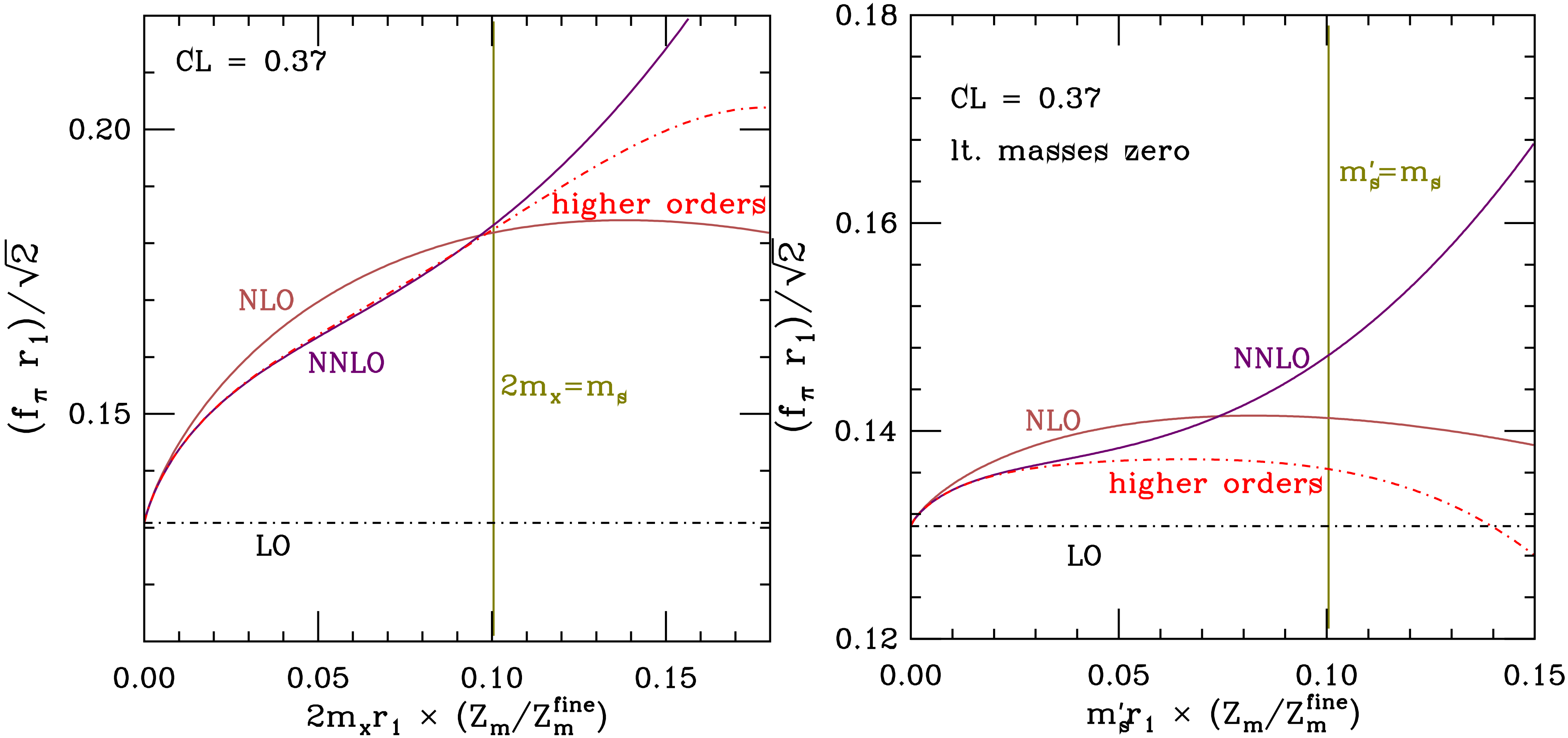




Delicate issues: same as before plus (iii) non-unitarity due to $a^{\rm stag} > 0$.

[MILC calculation continued]

Strong point of MILC is to have data with $m_s \ll m_s^{\text{phys}}$ to facilitate SU(3) limit !

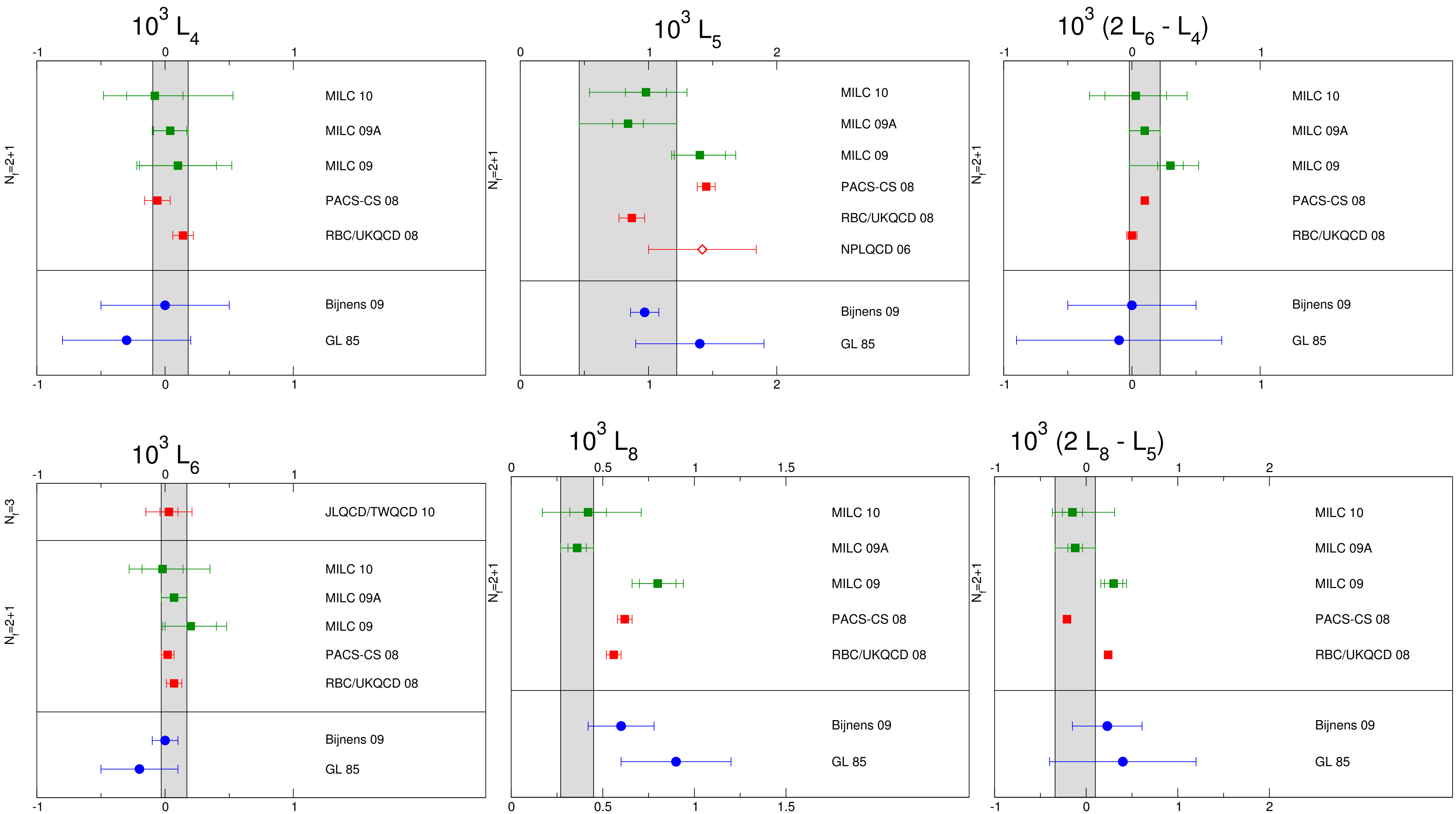


L : LO/NLO/NNLO-breakup of 3-flavor (degenerate) chiral behavior.

R: LO/NLO/NNLO-breakup of 2-flavor chiral limit as a function of m_s.

Issues with SU(3) ChPT (3): FLAG summary

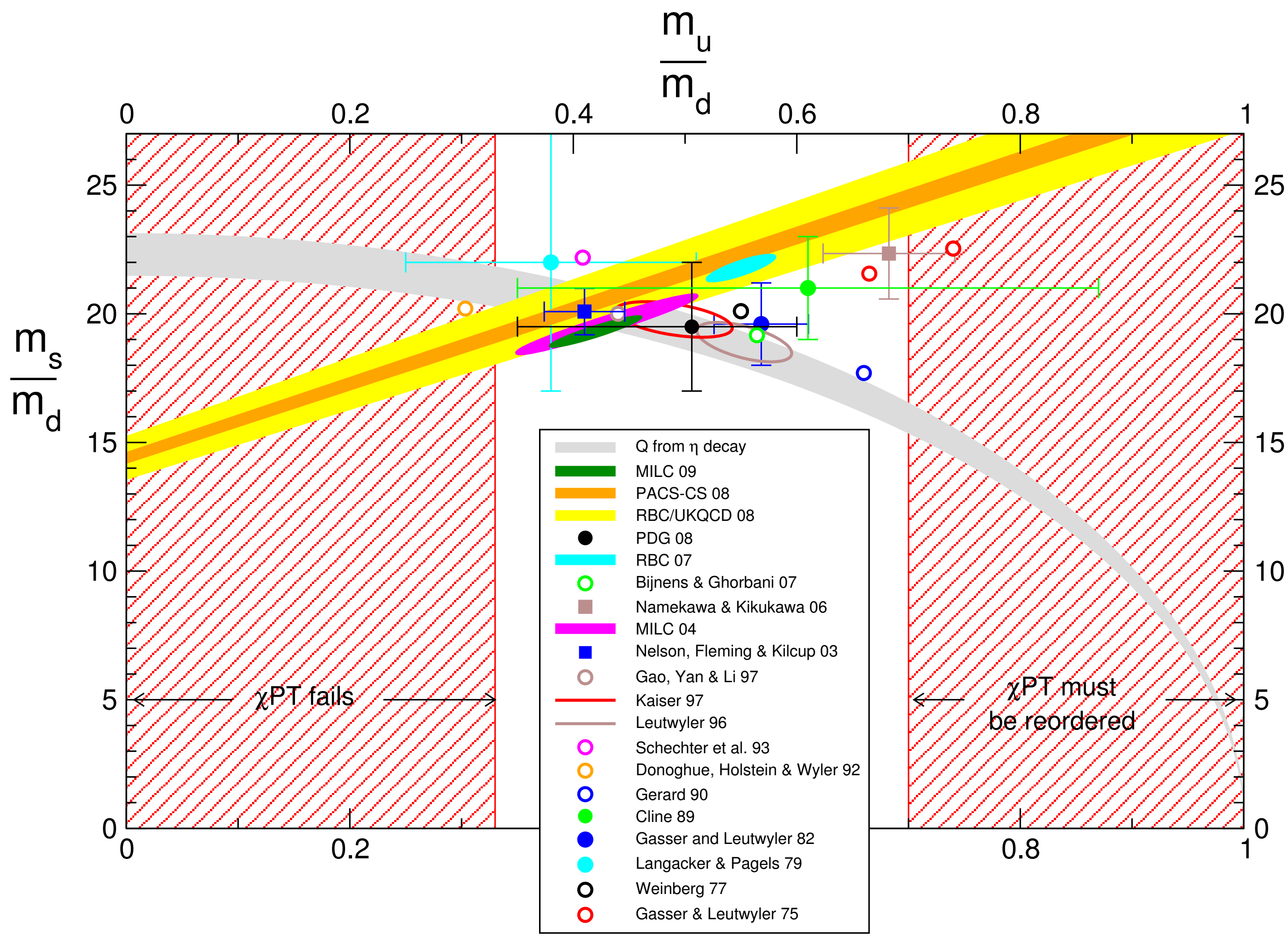
Colangelo *et al.* [FLAG consortium], Eur.Phys.J. C71 (2011) 1695 [arXiv:1011.4408]



$\longrightarrow$ Overall consistency (though mostly from $N_f = 2+1$ data with limited m_s range).

Potential failure/reordering of ChPT

Chiral expansion would make no sense if u-quark anywhere close to massless.
Chiral expansion needs to be reordered for u and d nearly degenerate (on rel. scale).
More precisely, ChPT+QED only adequate for $0.25 < m_u/m_d < 0.7$ [Leutwyler, CD'09].



Lattice excludes $m_u/m_d < 0.25$ by many sigmas [BMW, MILC, RM123, → A. Portelli].

Summary

- List of conditions which ChPT framework suitable to analyze which LQCD data.
- SU(2) framework is success story: varying m_{ud} around/above m_{ud}^{phys} yields $\bar{\ell}_3$, $\bar{\ell}_4$.
- SU(3) framework raises questions: perhaps m_s^{phys} too high for good convergence.
- Convergence issues pertinent to $m^{\mathrm{sea}} \neq m^{\mathrm{val}}$ and $a^{\mathrm{stag,tm}} > 0$ barely explored.
- Quark mass ratios $\frac{m_s}{m_{ud}}$, $\frac{m_u}{m_d}$ known to good accuracy $\longrightarrow$ no reorderings needed.