Light quark masses: from fiction to fact

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based on work with

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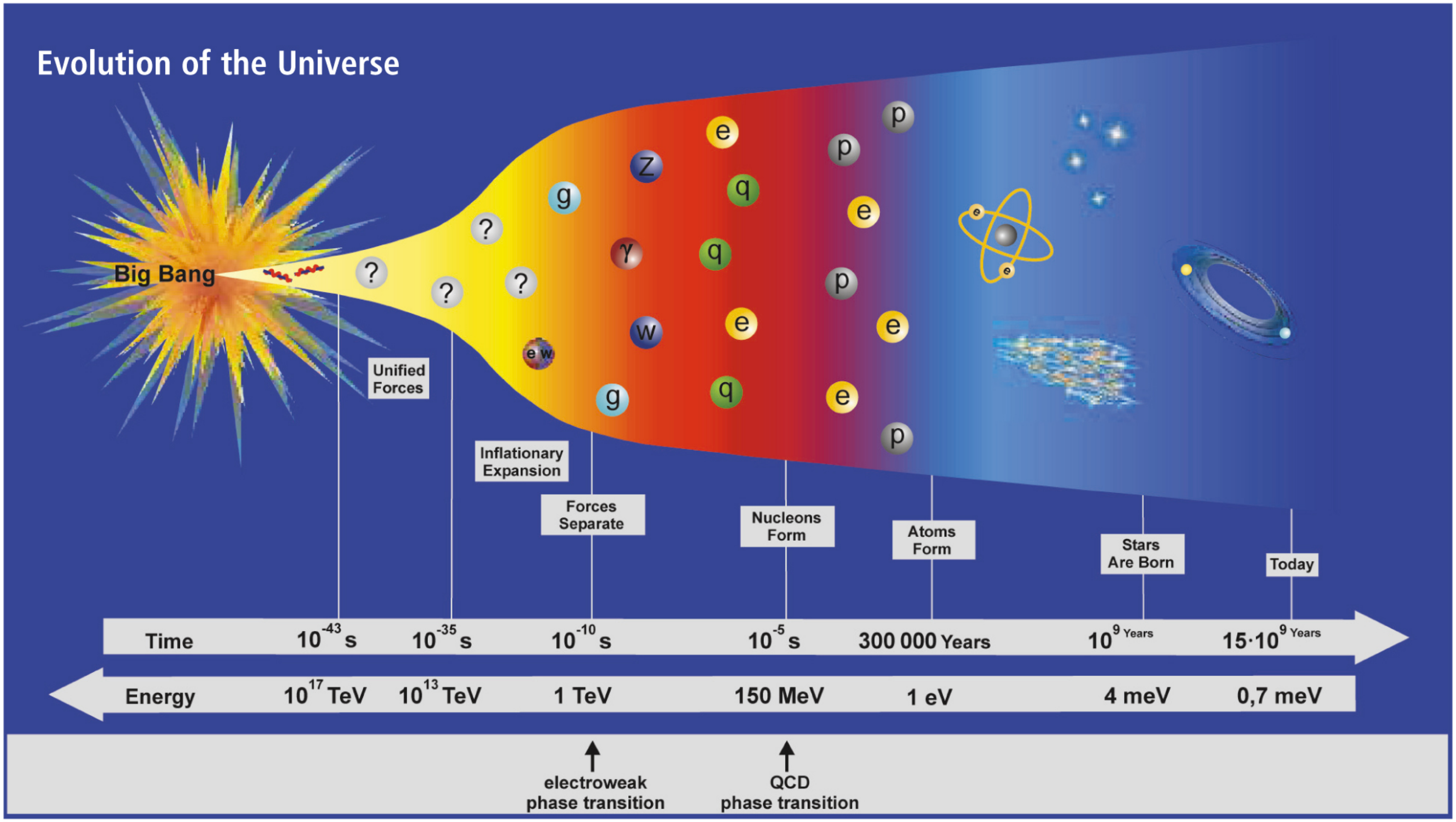
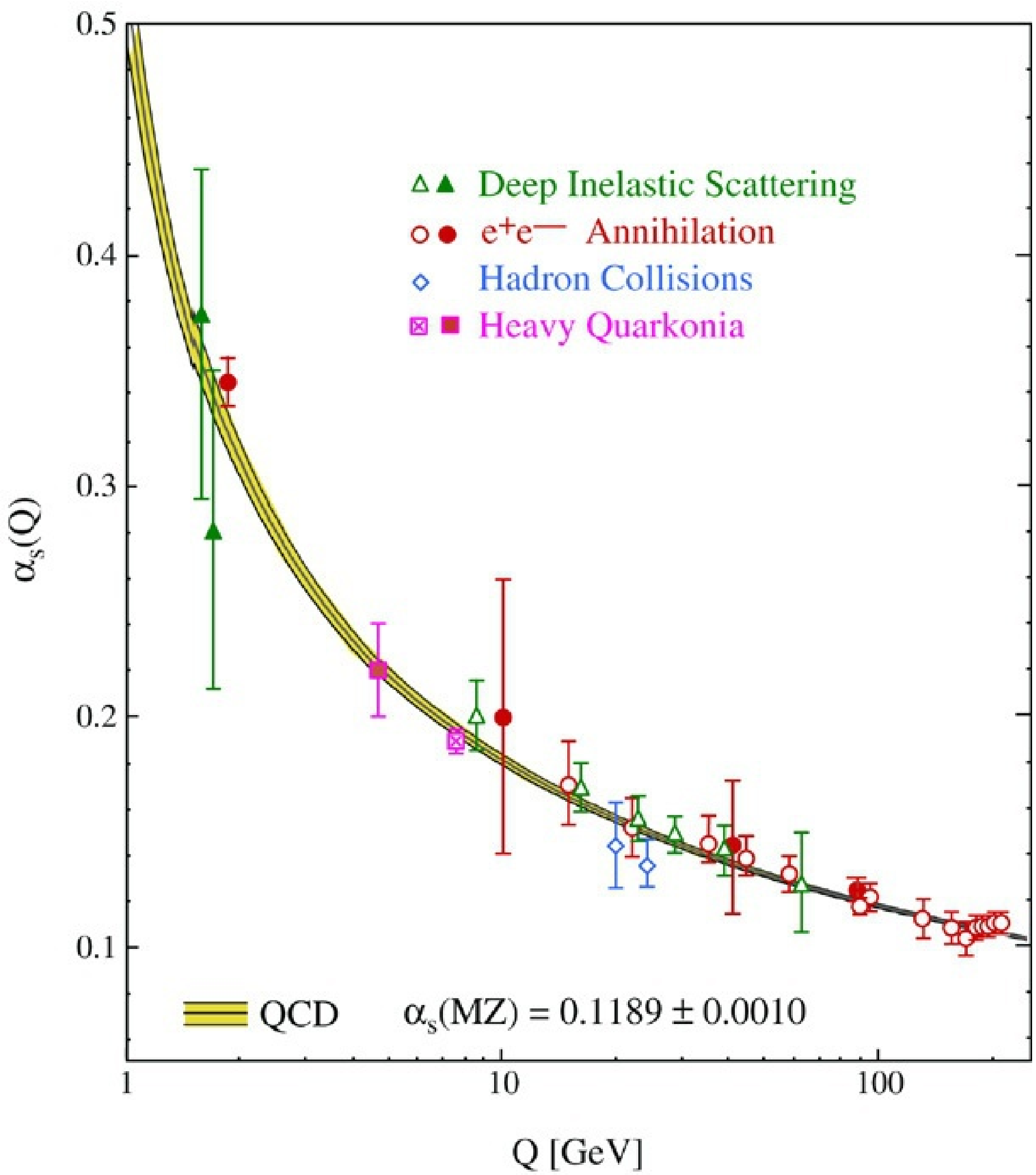
Origin of mass: EW versus QCD phase transition



- EW symmetry breaking (times Yukawa couplings) generates quark masses: $m_u = 2.4 \pm 0.7\,\mathrm{MeV}$, $m_d = 4.9 \pm 0.8\,\mathrm{MeV}$, $m_s = 105 \pm 25\,\mathrm{MeV}$ [PDG'10].
- QCD chiral/conformal symmetry breaking generates nucleon mass: $M_{p/n} \simeq 890\,\mathrm{MeV}$ at $m_{ud}=0$ (to be compared with $940\,\mathrm{MeV}$ at m_{ud}^{phys}).

QCD at high energies



Asymptotic freedom

[t'Hooft 1972, Gross-Wilczek/Politzer 1973]

$$\frac{\beta(\alpha)}{\alpha} = \frac{\mu}{\alpha}\frac{\partial\alpha}{\partial\mu} = \beta_1\alpha^1 + \beta_2\alpha^2 + \ldots$$

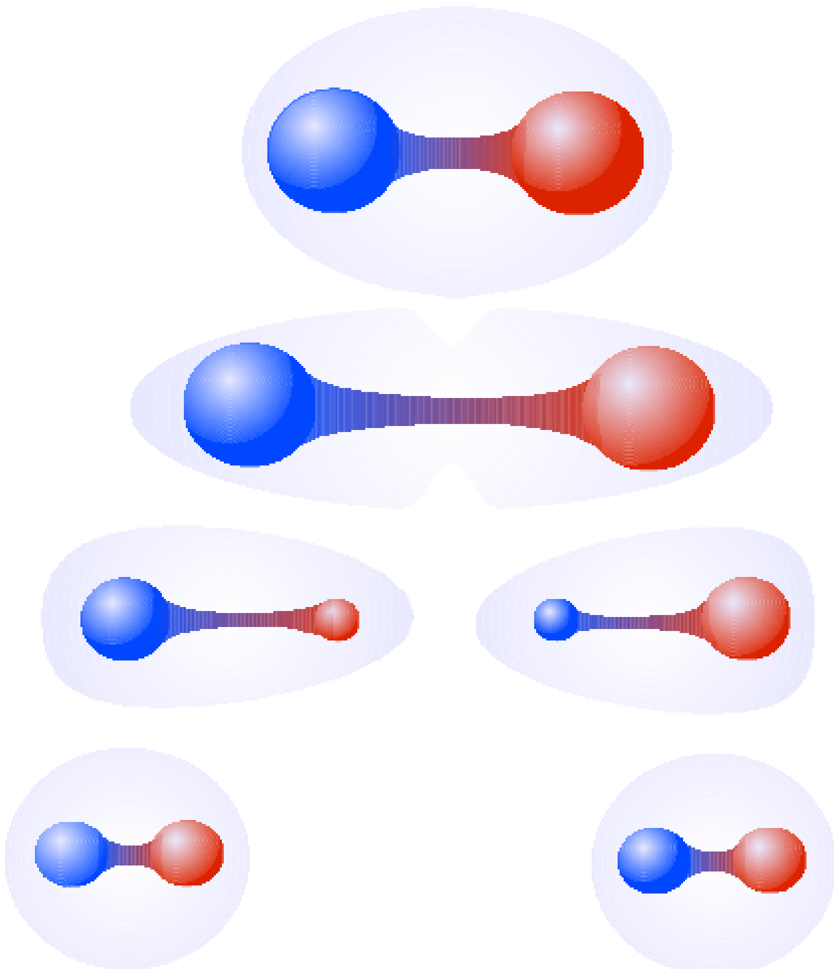
$$\beta_1 = (-11N_c + 2N_f)/(6\pi)$$

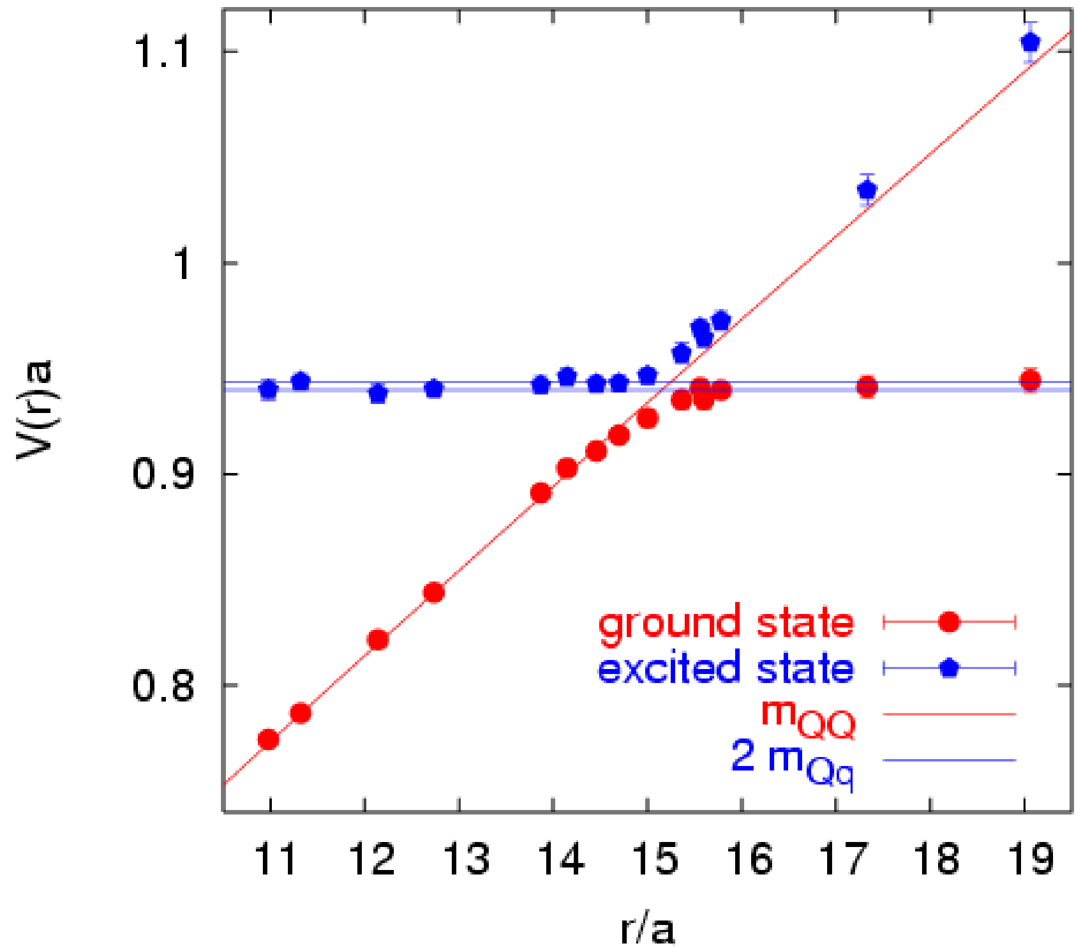
with $N_c = 3$ gives

$\beta_1 < 0$ for $N_f < 33/2$

- virtual gluons anti-screen, i.e. they make a static color source appear *stronger* at large distance.
- virtual quarks weaken this effect.

QCD at low energies

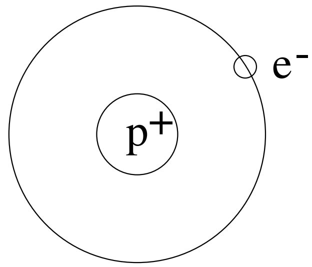




- In quenched QCD the $\bar{Q}Q$ potential keeps growing, $V(r)\!=\!\alpha/r\!+\!\mathrm{const}\!+\!\sigma r$.
- In full QCD it is energetically more favorable to pop a light $\bar{q}q$ pair out of the vacuum, $V(r)\!\leq\!\mathrm{const}$. Analysis with explicit $\bar{Q}q\bar{q}Q$ state: Bali et al., PRD 71, 114513 (2005).

Bound state dynamics in QED versus QCD

QED



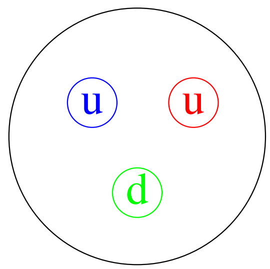
$M_e = 0.5$ MeV

$M_p = 938$ MeV

$E_{binding} = 13.6$ eV

Hydrogen Atom (EM force)

QCD



$M_u \sim 3$ MeV

$M_d \sim 6$ MeV

$M_p = 938$ MeV

Proton (Strong force)

Q0: What is the physical meaning of the "wrong sign" of the proton binding energy if *current quark masses* are used ?

Q1: Do we understand strong dynamics sufficiently well as to postdict the mass of the proton ?

Q2: If so, can we turn the calculation around and determine $m_{ud} = (m_u + m_d)/2$ from first principles ?

Talk outline

- Information from Chiral Perturbation Theory (XPT)
- Information from Sum Rules (SR)

- Lattice QCD: $m_q^{\mathrm{bare}} \longleftrightarrow M_{\pi,K,\Omega}$
- Commercial: Why it helps to bracket m_q^{phys}
- Lattice QCD: $m_q^{\mathrm{bare}} \longleftrightarrow m_q^{\mathrm{SF/RI}} \longleftrightarrow m_q^{\overline{\mathrm{MS}}}$
- Commercial: Why it helps to have contact with PT
- Lattice QCD: shortcut via m_{ud}/m_s, m_s/m_c and m_c from elsewhere

- Summary: reference to FLAG review
- Outlook: $N_f = 1{+}1{+}1{+}1$ simulations with electromagnetism

Chiral Perturbation Theory (1): framework

SU(3) Lagrangian with quark masses set to zero

$$L_{\rm QCD} = -\frac{1}{4}{\rm Tr}G_{\mu\nu}G^{\mu\nu} + \bar{q}_L {\rm i}\not{D} q_L + \bar{q}_R {\rm i}\not{D} q_R$$

with $q = (u,d,s)^{\rm transp}$ and $q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$ exposes large symmetry group $SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$. One factor breaks down spontaneously as $SU(3)_L \times SU(3)_R \longrightarrow SU(3)_V$, which gives rise to an octet of Goldstone bosons.

General mass expansion for a particle P

$$M^2 = M_0^2 + (m_u{+}m_d)\langle P|\bar{q}q|P\rangle + O(m_q^2)$$

simplifies for a (pseudo-) Goldstone boson as

$$M^2 = -(m_u{+}m_d)\frac{1}{F^2}\langle 0|\bar{q}q|0\rangle + O(m_q^2)$$

where we have used the Ward identity $\langle \pi|\bar{q}q|\pi\rangle = -\frac{1}{F^2}\langle 0|\bar{q}q|0\rangle \equiv B$.

Chiral Perturbation Theory (2): quark mass ratios

Consider whole pseudoscalar octet:

$$M_{\pi^+}^2 = B_0(m_u+m_d)\;, \quad M_{K^+}^2 = B_0(m_u+m_s)\;, \quad M_{K^0}^2 = B_0(m_d+m_s)$$

Quark mass ratios [necessary, since only $B_0 m_q$ shows up]:

$$\frac{m_u}{m_d} = \frac{M_{\pi^+}^2 - M_{K^0}^2 + M_{K^+}^2}{M_{\pi^+}^2 + M_{K^0}^2 - M_{K^+}^2} \simeq 0.67$$

$$\frac{m_s}{m_d} = \frac{M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} \simeq 20.0$$

Combine with SU(6) based estimate $m_{ud} \equiv (m_u+m_d)/2 = 5.4\,\mathrm{MeV}$ [Leutwyler'75]

$$m_u \sim 4\,\mathrm{MeV}\;, \quad m_d \sim 6\,\mathrm{MeV}\;, \quad m_s \sim 135\,\mathrm{MeV}\;.$$

Improvement from including electromagnetic corrections (Dashen's theorem).

Surprise from pushing to higher order in the chiral expansion (KM ambiguity).

Analytical Sum Rules (1): framework

2-point correlator of weak currents $L^\mu = \bar{u}\gamma^\mu(1-\gamma_5)d_\theta$ with Cabibbo rotated d_θ is

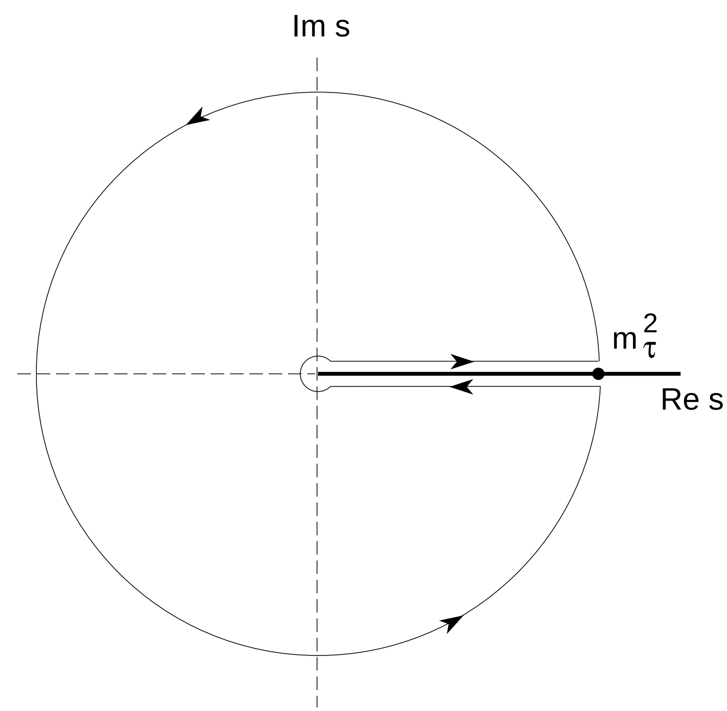
$$\begin{aligned}\Pi_L^{\mu\nu}(q) &= \mathrm{i}\int d^4x\; e^{\mathrm{i}qx}\langle 0|T\{L^\mu(x)L^\nu(0)^\dagger|0\rangle \\ &= (-g^{\mu\nu}q^2+q^\mu q^\nu)\Pi_L^{(1)}(q^2)+q^\mu q^\nu\Pi_L^{(0)}(q^2)\end{aligned}$$

Physically interesting quantities are related to an integration of the type $\int_0^{m_\tau^2} ds$ of $\mathrm{Im}\Pi^{(n)}(s)$ with various weight functions.

Trade integral along the cut of

$$\mathrm{Im}\Pi_L^n(s) = \frac{1}{2\mathrm{i}}\Big[\Pi_L^n(s+\mathrm{i}\epsilon)-\Pi_L^n(s-\mathrm{i}\epsilon)\Big]$$

for an integral along the circle $|s| = m_\tau^2$.

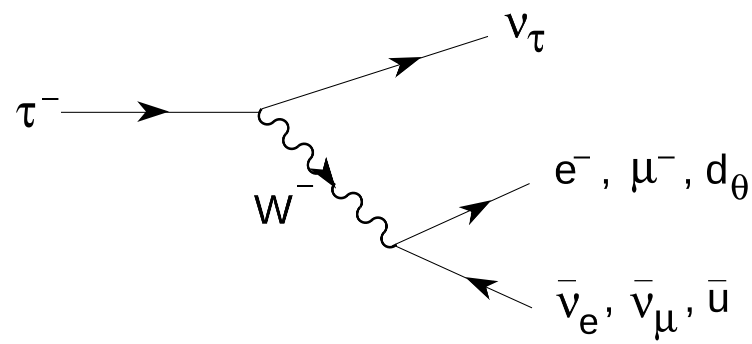


Analytical Sum Rules (2): quark mass values

Old sum rule results for $m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$ usually clustered around $125\,\mathrm{MeV}$.

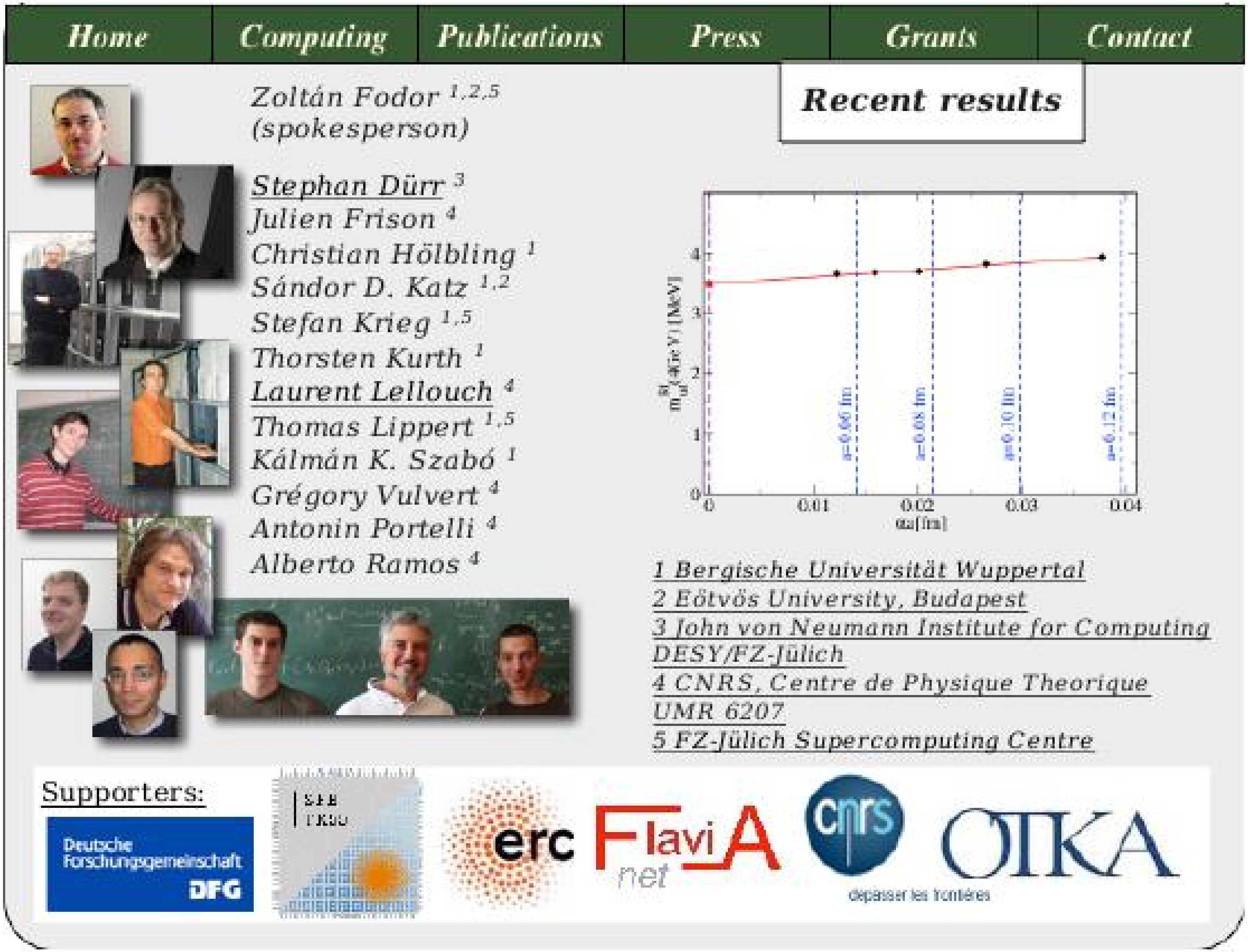
Summary of semi-recent results for $m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$ in MeV based on τ data:

Jamin et al. (02)	99 ± 16
Kambor Maltman (02)	100 ± 12
Gamiz et al (03)	103 ± 17
Jamin et al. (05)	81 ± 22
Gorbunov Pivovarov (05)	125 ± 28
Baikov et al. (05)	96 ± 19
Narison (05)	89 ± 25



The first set is mostly based on ALEPH data.
The second one includes data from OPAL and CLEO.

Budapest-Marseille-Wuppertal collaboration



Lattice QCD (1): combined UV/IR regulator

Elementary degrees of freedom are quarks and gluons, transforming in the fundamental representation of $SU(3)$ [Fritzsch, Gell-Mann and Leutwyler (1973)]. In euclidean space:

$$\mathcal{L}_{\mathrm{QCD}} = \frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}) + \sum_{i=1}^{N_f} \bar{q}^{(i)}(\not{D}+m^{(i)})q^{(i)} + \mathrm{i}\theta\frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}(F_{\mu\nu}F_{\rho\sigma})$$

- QCD must be regulated both in the UV and in the IR to make it well-defined; such a regulator is a necessary ingredient in a QCD calculation.
- The lattice does this by $a>0$ and $V=L^4<\infty$, but other options are possible. In fact, each gauge/fermion action is a different regulator.
- The extrapolations $a\to 0$ and $V\to\infty$ are performed in the resulting observables.
- The result is independent of the action, thanks to universality (spin syst, RG, FP).

$\Longrightarrow$ Lattice discretization is not an approximation to continuous space-time, but (generically) an *unavoidable interim part* of the definition of QCD !

$\Longrightarrow$ Does this *Lagrangian-regulator-extrapolation package* explain confinement, chiral/conformal symmetry breaking, hadron spectrum, ... ?

Lattice QCD (2): crash course

- **QFT on the lattice**

$Z=\int D\phi\, e^{-S[\phi]},\ \ S[\phi]=\frac{1}{2}(\nabla\phi)^2+\frac{m}{2}\phi^2+\ldots,\ \ D\phi$ means $-\infty<\phi(x)<\infty$ for each x

- **Gluons on the lattice**

$$U_\mu(x)U_\nu(x+a\hat{\mu})-U_\nu(x)U_\mu(x+a\hat{\nu})=\mathrm{i}a^2F_{\mu\nu}(x)+O(a^3)$$
$$F_{\mu\nu}(x)=\partial_\mu A_\nu(x)-\partial_\nu A_\mu(x)+ig[A_\mu(x),A_\nu(x)]$$
$$S[U]=\beta\sum_\Box\{1-\tfrac{1}{3}\mathrm{ReTr}(U_\Box)\}\ \rightarrow\ \frac{a^4}{g^2}\sum_{x,\mu<\nu}\mathrm{Tr}(F_{\mu\nu}(x)^2)$$

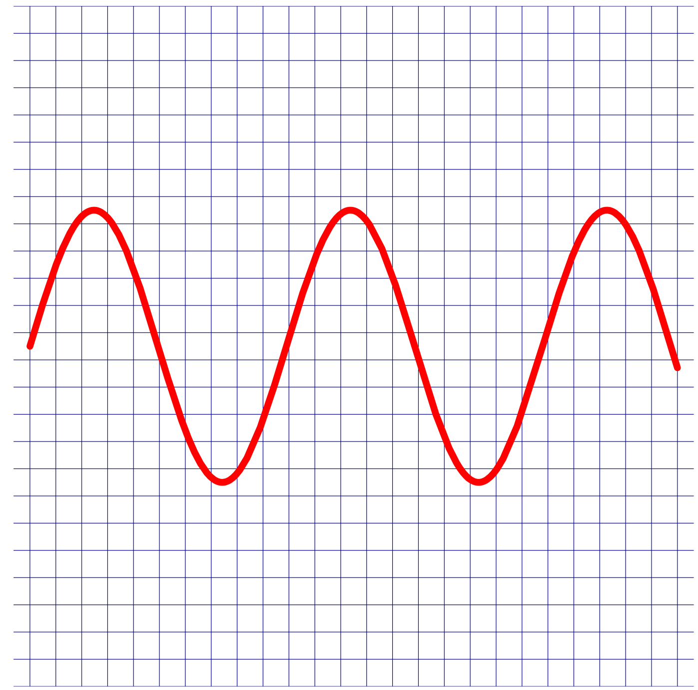
- **Quarks on the lattice**

$S[U]\ \rightarrow\ S[U]-\log(\det(D[U]))$, still integrate over $SU(3)$ for each link

- **Computation overview**

1. Generate configurations U distributed according to $p[U]=e^{-S[U]}\det^{N_f}(D[U])$.
2. Solve $D[U]x=b$, build propagators to measure $C(t)$ for various states.
3. Use PT/SF/RI to renormalize/match to continuum schemes (e.g. $\overline{\mathrm{MS}}$).
4. Use effective field theories to extrapolate $a\to 0$, $L\to\infty$, maybe $m_q\to m_q^{\mathrm{phys}}$.

Lattice QCD (3): scale hierarchies



typical spacing: $0.05\,\mathrm{fm} \le a \le 0.20\,\mathrm{fm}$

$1\,\mathrm{GeV} \le a^{-1} \le 4\,\mathrm{GeV}$

typical length: $2\,\mathrm{fm} \le L \le 6\,\mathrm{fm}$

require: $am_q \ll 1$ and $aM_{\mathrm{had}} \ll 1$

require: $M_\pi L > 4$ [note $4/M_\pi^{\mathrm{phys}} \simeq 5.8\,\mathrm{fm}$]

u	c	(t)
d	s	b
extrapolate	work at/near	extrapolate
$m_{ud} \searrow m_{ud}^{\mathrm{phys}}$	m_s^{phys}, m_c^{phys}	$m_b \nearrow m_b^{\mathrm{phys}}$
or use m_{ud}^{phys}	values*	$m_\infty \searrow m_b^{\mathrm{phys}}$

Asterisk: Tune to appropriate (bare) am_q for each lattice spacing and each flavor.
In QCD with N_f quarks, $N_f{+}1$ observables used to determine quark masses and scale.

Lattice QCD spectroscopy (1)

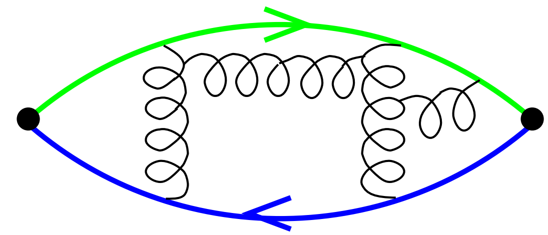
Hadronic correlator in $N_f \geq 2$ QCD: $C(t) = \int d^4x\, C(t,\mathbf{x})\, e^{i\mathbf{px}}$ with

$$C(x) = \langle O(x)\, O(0)^\dagger \rangle = \frac{1}{Z}\int DU\, D\bar{q} Dq\;\; O(x)\, O(0)^\dagger\;\; e^{-S_G - S_F}$$

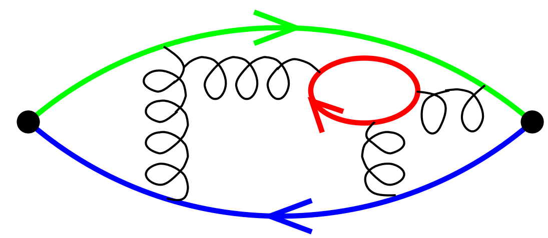
where $O(x) = \bar{d}(x)\Gamma u(x)$ and $\Gamma = \gamma_5, \gamma_4\gamma_5$ for $\pi^\pm$ and
$S_G = \beta \sum (1 - \frac{1}{3}\mathrm{Re}\mathrm{Tr}\, U_{\mu\nu}(x))$, $S_F = \sum \bar{q}(D+m)q$

$$\langle \bar{d}(x)\Gamma_1 u(x)\; \bar{u}(0)\Gamma_2 d(0)\rangle = \frac{1}{Z}\int DU\;\det(D+m)^{N_f}\; e^{-S_G}$$

$$\times\, \mathrm{Tr}\Big\{\Gamma_1 (D+m)^{-1}_{x0}\, \Gamma_2\, \underbrace{(D+m)^{-1}_{0x}}_{\gamma_5[(D+m)^{-1}_{x0}]^\dagger\gamma_5}\Big\}$$



(A) Quenched QCD: quark loops neglected

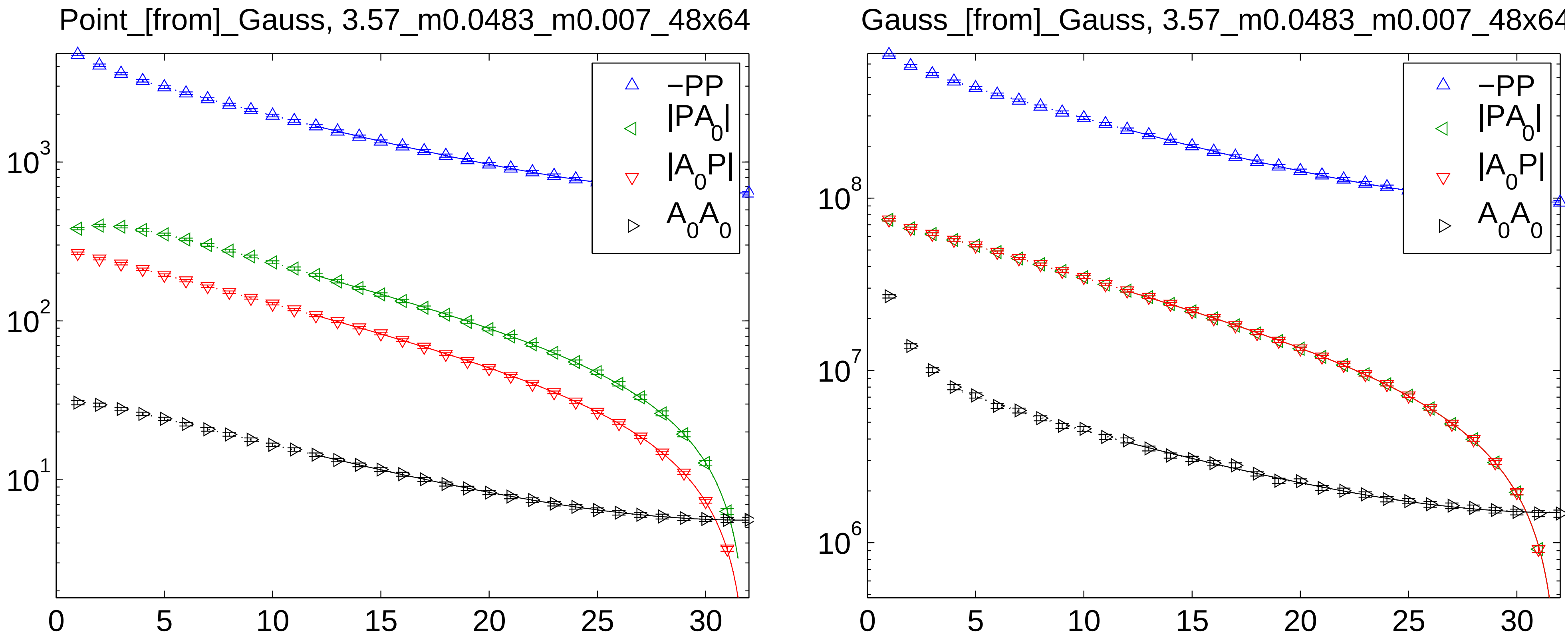


(B) Full QCD

- Choose $m_u = m_d$ to save CPU time, since isospin $SU(2)$ is a good symmetry.
- In principle $m_{\mathrm{valence}} = m_{\mathrm{sea}}$, but often additional valence quark masses to broaden data base. Note that “partially quenched QCD” is an *extension* of “full QCD”.
- $(D+m)^{-1}_{x0}$ for all x amounts to 12 *columns* (with spinor and color) of the inverse.

Lattice QCD spectroscopy (2)

Excellent data quality even on our lightest ensemble ($M_\pi \simeq 190\,\mathrm{MeV}$ and $L \simeq 4.0\,\mathrm{fm}$):



$\cosh(.)/\sinh(.)$ for $-PP, |PA_0|, |A_0P|, A_0A_0$ with Gauss source and local/Gauss sink

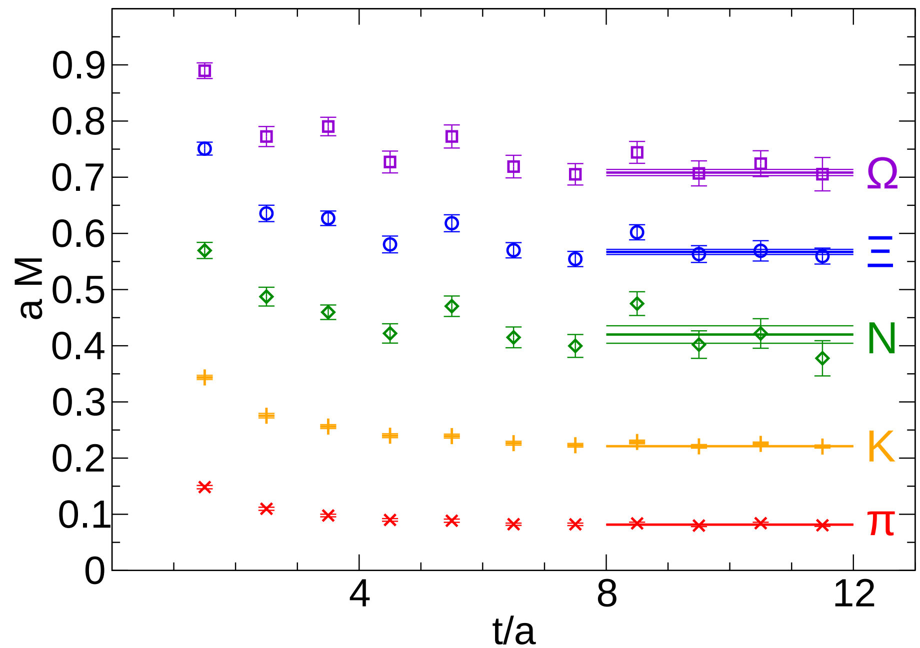
$C_{Xx,Yy}(t) = c_0 e^{-M_0 t} \pm c_0 e^{-M_0(T-t)} + \ldots$ with $X, Y \in \{P, A_0\}$ and $x, y \in \{\mathrm{loc}, \mathrm{gau}\}$

$\longrightarrow$ $c_0 = G\tilde{G}/M_0, G\tilde{F}, F\tilde{G}, F\tilde{F}M_0$ (left) and $c_0 = \tilde{G}\tilde{G}/M_0, \tilde{G}\tilde{F}, \tilde{F}\tilde{G}, \tilde{F}\tilde{F}M_0$ (right)

$\longrightarrow$ combined 1-state fit of 8 correlators with 5 parameters yields $M_\pi, F_\pi, m_{\mathrm{PCAC}}$

Lattice QCD spectroscopy (3)

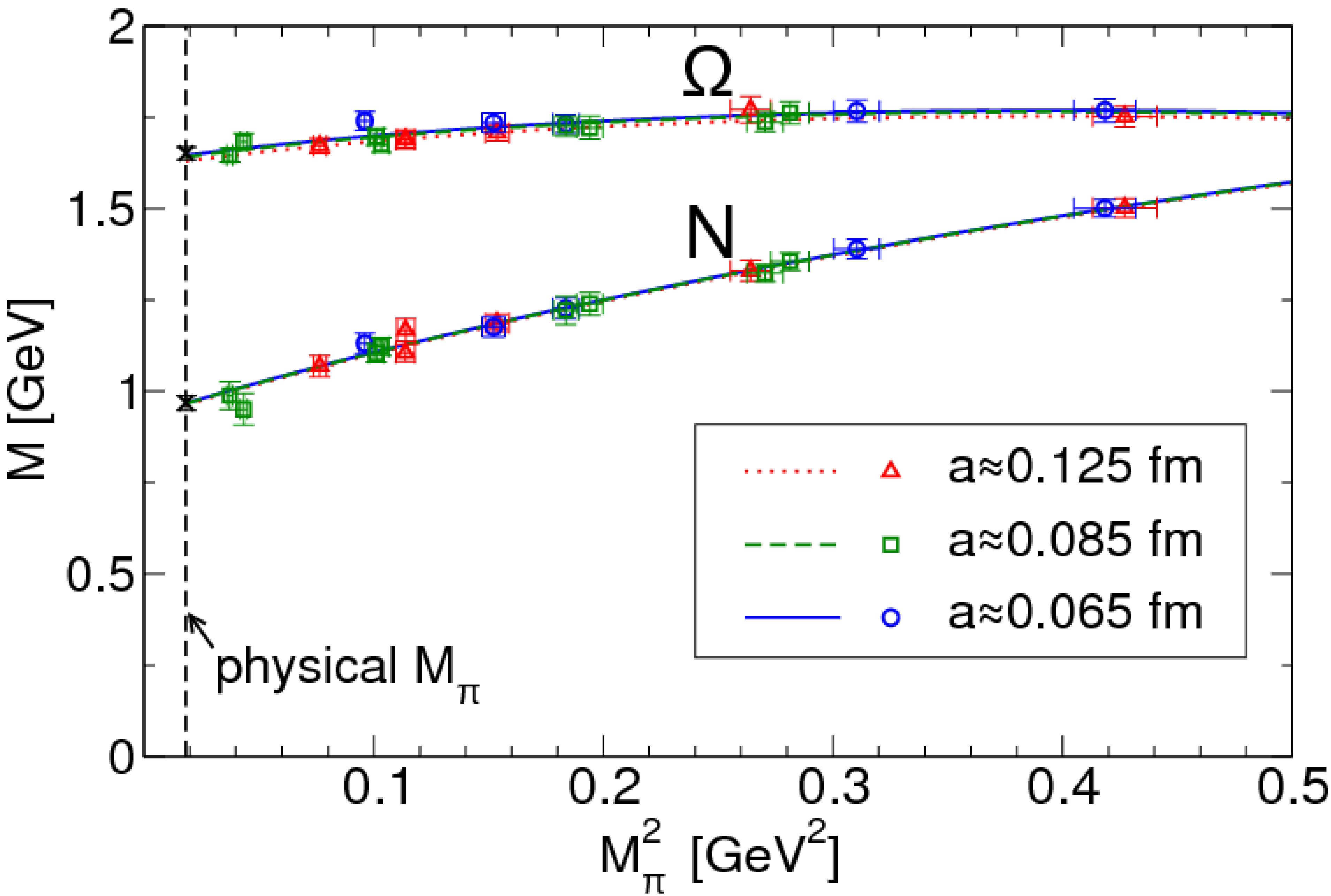
With similar techniques for other channels we find in each run
aM_π, aM_K, aM_ρ, aM_{K^*}, aM_N, aM_Σ, aM_Ξ, aM_Λ, aM_Δ, aM_{Σ^*}, aM_{Ξ^*}, aM_Ω.



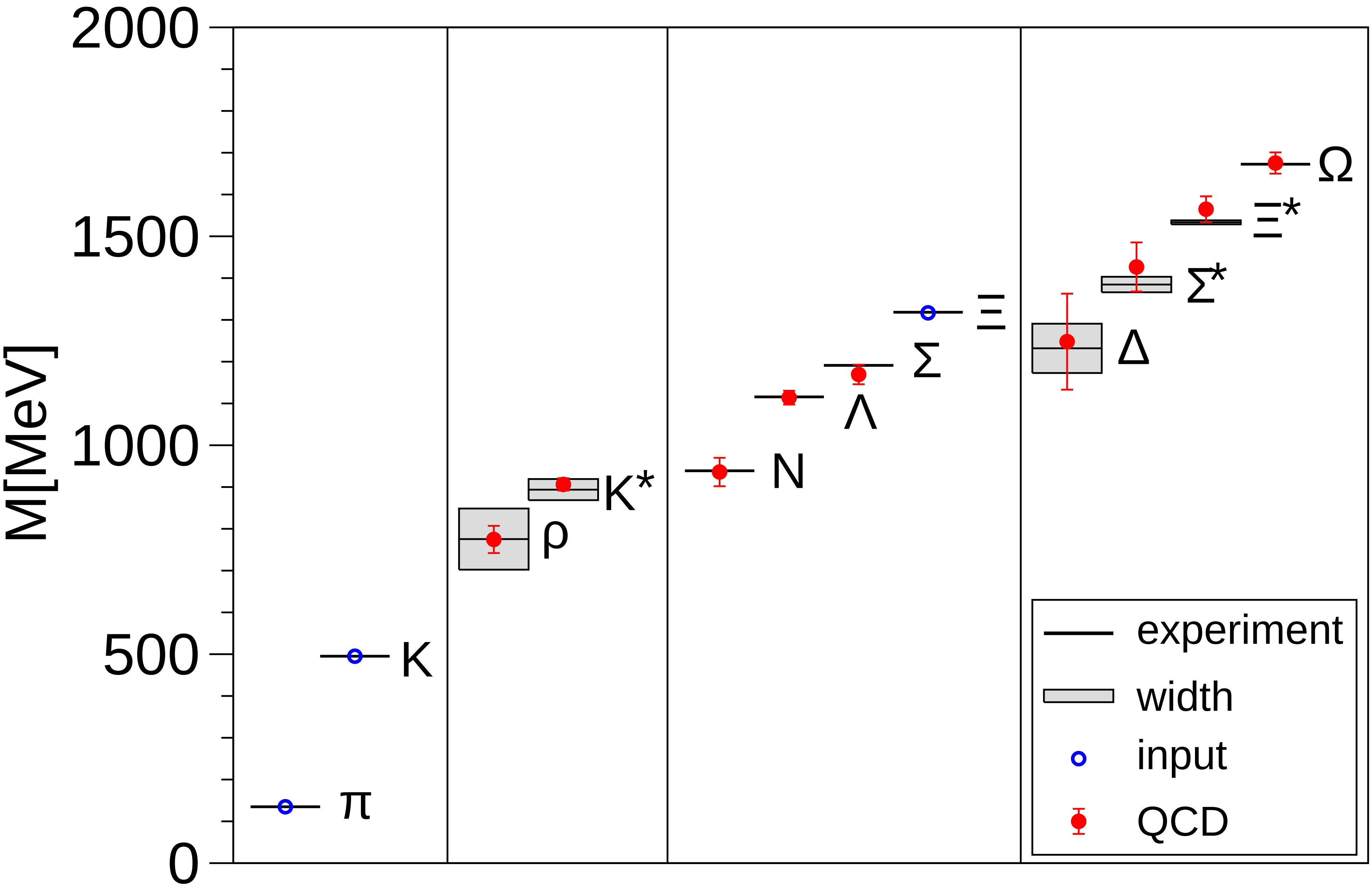
Cost growth (Lattice 2001, "Berlin wall phenomenon") recently tamed [in two parts]:

$a \to 0$	"continuum limit"	cost $\propto (1/a)^{4-6}$
$V \to \infty$	"infinite volume limit"	cost $\propto V^{5/4}$ with HMC
$m_{ud} \to m_{ud}^{\text{phys}}$	"chiral/physical limit"	cost $\propto (1/m)^{1-2}$ with tricks
δ(observable) $\to 0$	"reduce statistical error"	cost $\propto \delta^{-2}$

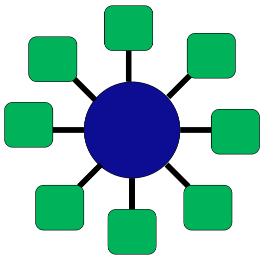
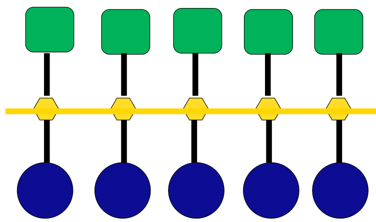
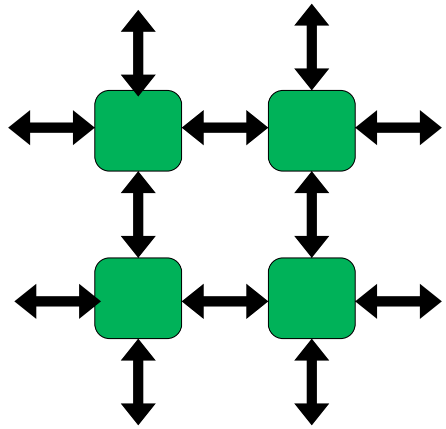
Lattice QCD spectroscopy (4)



Final result: BMW collaboration, Science 322, 1224 (2008)



Technical interlude: machine details


"JUGENE" [IBM BG/P]	02/2008 - 02/2009	06/2009 - ...
processor type	32-bit PowerPC 450 core 850 MHz	(3.4 Gflops each)
compute node	4-way SMP processor	
racks, nodes, processors	16, 16'384, 65'536	72, 73'728, 294'912
memory	2 GB per node, aggregate 32 TB	aggregate 144 TB
performance (peak/Lapack)	223/180 Teraflops [double prec.]	1/0.825 Petaflops
power consumption	$<$40 kW/rack, aggregate 0.5 MW	2.2 Megawatt
network topology	3D torus among compute nodes (plus global tree collective network, plus ethernet admin network)	
network latency	160 nsec (light travels 48 meters)	
network bandwidth	5.1 Gigabyte/s	

Lattice Perturbation Theory

$$\langle .|O_j^{\rm cont}(\mu)|.\rangle = \sum_k Z_{jk}(a\mu)\langle .|O_k^{\rm latt}(a)|.\rangle$$

$$Z_{jk}(a\mu) = \delta_{jk} - \frac{g_0^2}{16\pi^2}(\Delta_{jk}^{\rm latt} - \Delta_{jk}^{\rm cont}) = \delta_{jk} - \frac{g_0^2}{16\pi^2}C_F z_{jk}$$

$$Z_S(a\mu) = 1 - \frac{g_0^2}{4\pi^2}\Big[\frac{z_S}{3} - \log(a^2\mu^2)\Big] \qquad Z_V = 1 - \frac{g_0^2}{12\pi^2}z_V$$

$$Z_P(a\mu) = 1 - \frac{g_0^2}{4\pi^2}\Big[\frac{z_P}{3} - \log(a^2\mu^2)\Big] \qquad Z_A = 1 - \frac{g_0^2}{12\pi^2}z_A$$

Generically $[z_P - z_S]/2 = z_V - z_A$, and for a chiral action either side vanishes.

Generically 1-loop LPT would yield results with leading cut-off effects $O(\alpha a)$, in line with the leading effects from our action, but the general hope/belief is that with non-perturbative improvement (in RI) the Symanzik scaling window is larger.

2 HEX study: ensemble overview

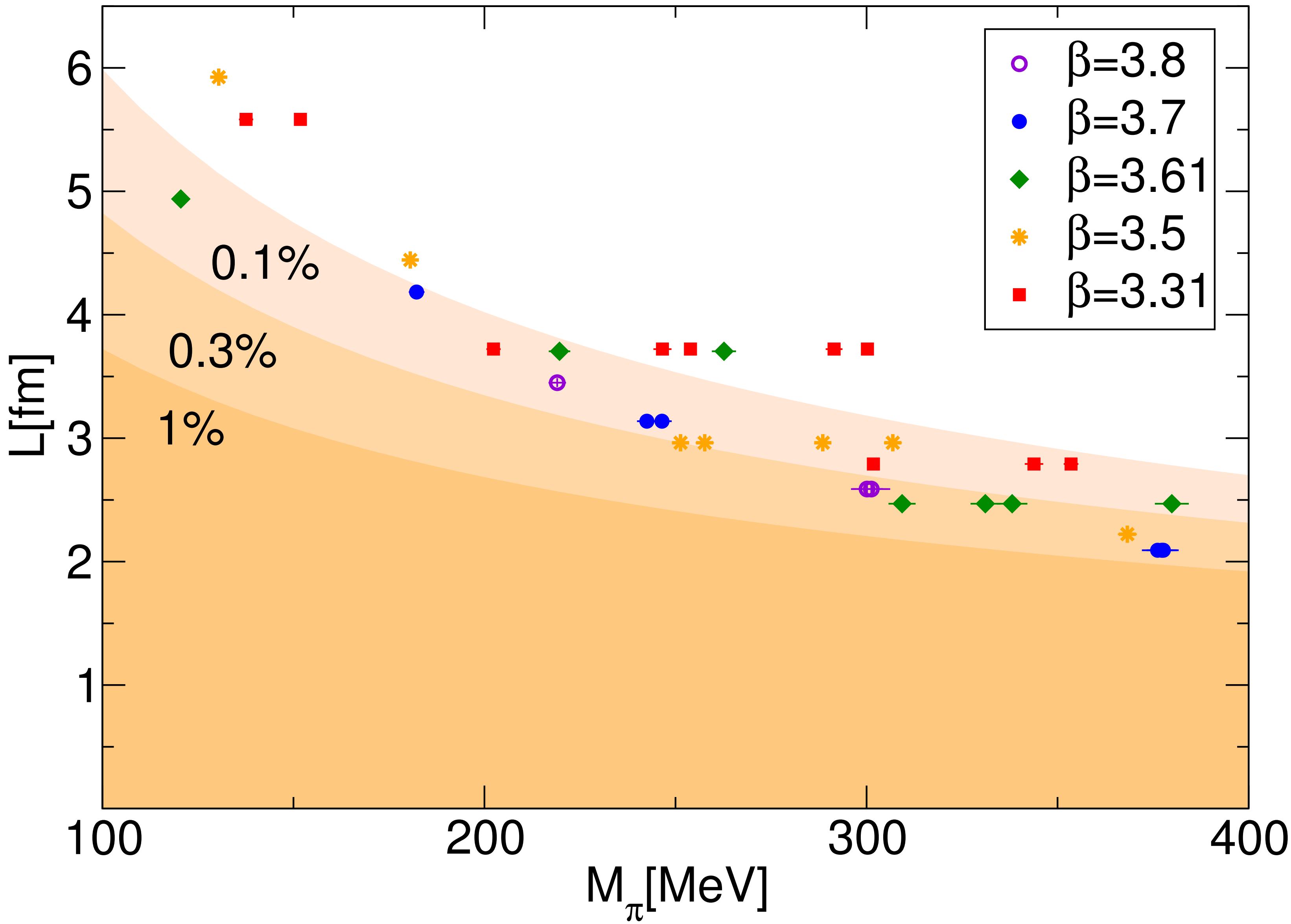
- Mass-independent scale setting [i.e. per β] in $N_f=2+1$ QCD at the point where M_π/M_Ω and M_K/M_Ω assume their physical values.

β	a[fm]	a^{-1}[GeV]	$\#(m_{ud},m_s)$
3.31	0.116	1.697(06)	11
3.5	0.093	2.131(13)	12
3.61	0.077	2.561(26)	9
3.7	0.065	3.026(27)	9
3.8	0.054	3.662(35)	6

- Bare masses (via ratio/difference method) on all $N_f=2+1$ ensembles.
- Additional $N_f=3$ ensembles at same β values for RI renormalization, with point-by-point (in p^2) chiral extrapolation of renormalization factors.
- Combined continuum extrapolation and interpolation of the renormalized m_q to the physical point $[M_\pi=135\,\mathrm{MeV}, M_K=495\,\mathrm{MeV}]$ on the former ensembles.

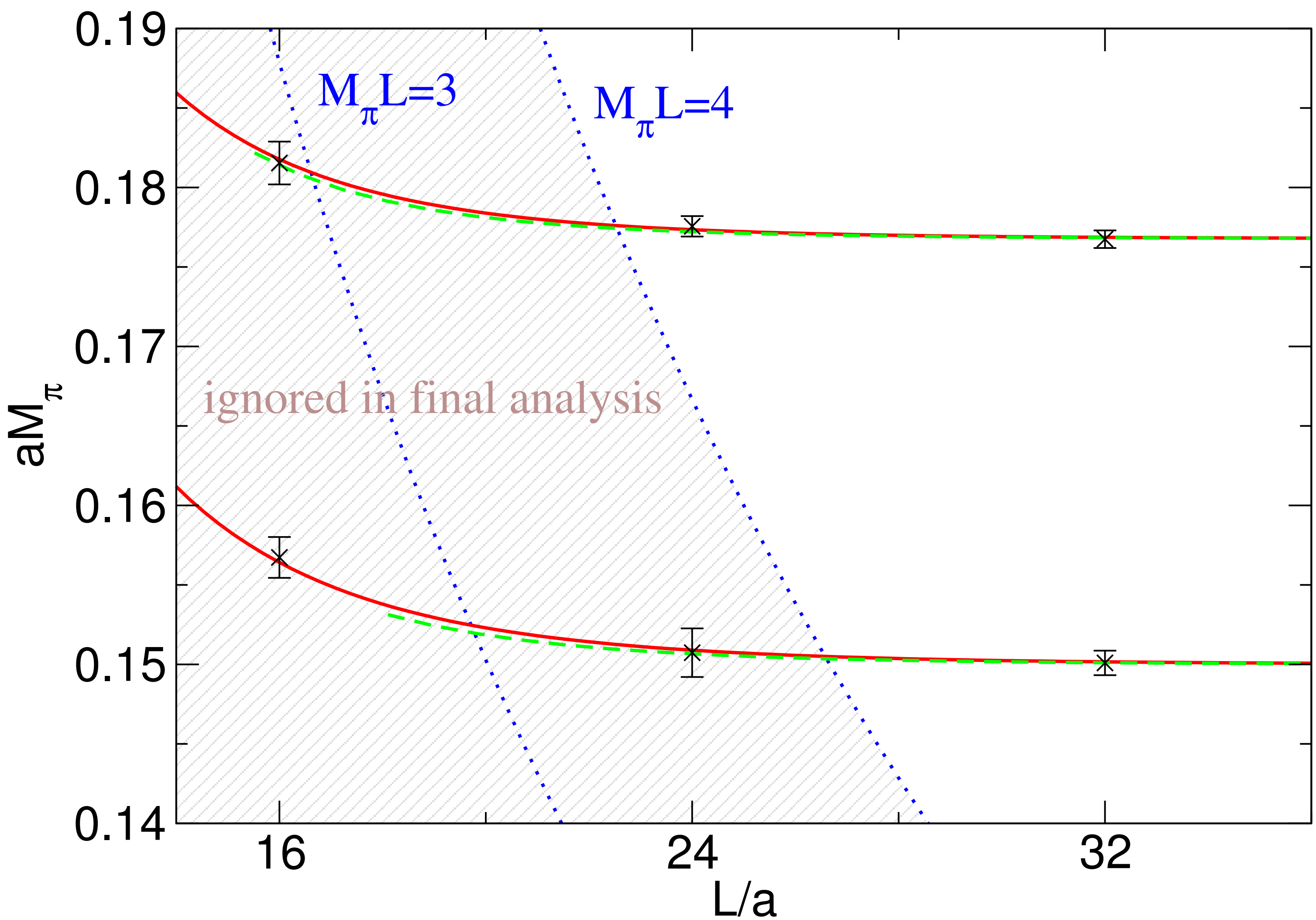
  $\longrightarrow$ *first study with Wilson-type quarks to reach physical mass point* !

2 HEX study: $N_f = 2+1$ simulation landscape



⟶ we can *interpolate* to $M_\pi^{\text{phys}} \simeq 135\,\text{MeV}$ at 3 out of 5 lattice spacings

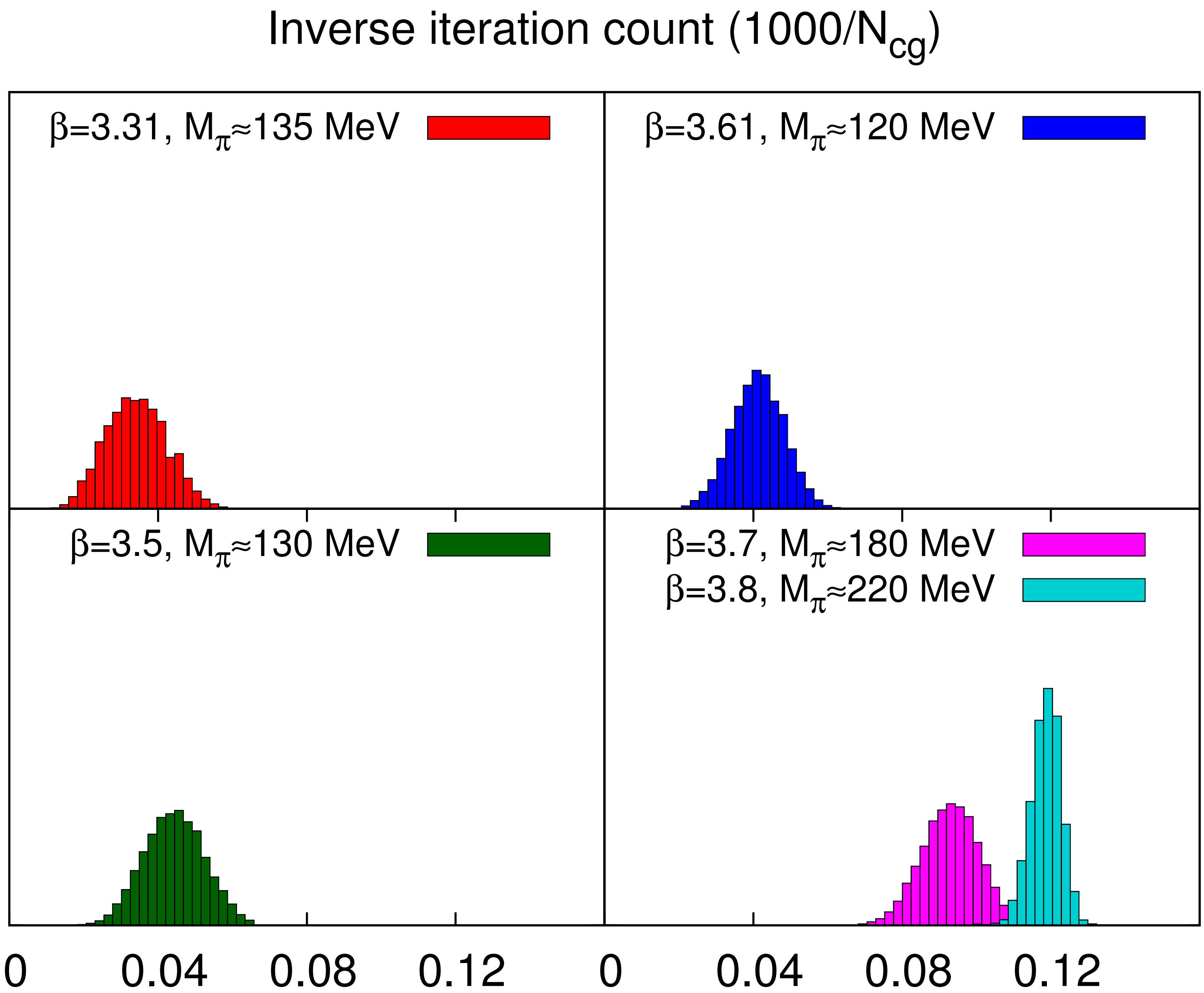
2 HEX study: $N_f=2+1$ finite volume corrections



Relative finite-volume corrections for masses and decay constants die out exponentially fast for large $M_\pi L$ [Lüscher'85].

Results at approximate 3-loop (M_π) and 2-loop (F_π) order are found in [CDH'05].

Algorithmic challenges: $1/n_{\mathrm{CG}}$ count



VWI versus AWI definition of quark masses

Bare Wilson mass undergoes additive and multiplicative renormalization:

$$m^{\mathrm{VWI}} = \frac{1}{Z_S}\left[1-\frac{1}{2}b_S a m^{\mathrm{W}} + O(a^2)\right] m^{\mathrm{W}} \qquad \text{where} \qquad m^{\mathrm{W}} = m^{\mathrm{bare}} - m^{\mathrm{crit}}$$

$Z_S = Z_S(\mu)$ is the lattice-to-continuum "renormalization" (matching) factor.

Alternatively one may use the axial Ward identity:

$$m_1^{\mathrm{PCAC}} + m_2^{\mathrm{PCAC}} = \frac{\sum_{\vec{x}} \langle \bar{\partial}_\mu [A_\mu(x) + a c_A \bar{\partial}_\mu P(x)] O(0) \rangle}{\sum_{\vec{x}} \langle P(x) O(0) \rangle}$$

A_μ and P denote the axial current and the pseudoscalar density.
O is an arbitrary operator which couples to the meson (usually $O = P$).
$\bar{\partial}_\mu \phi(x) = [\phi(x+a\hat{\mu}) - \phi(x-a\hat{\mu})]/(2a)$ is the symmetric derivative.

$$m^{\mathrm{AWI}} = \frac{Z_A}{Z_P} \frac{1 + b_A a m^{\mathrm{W}} + O(a^2)}{1 + b_P a m^{\mathrm{W}} + O(a^2)} m^{\mathrm{PCAC}}$$

Z_A and $Z_P = Z_P(\mu)$ are lattice-to-continuum "renormalization" (matching) factors.

Ratio-difference method for quark masses

- It is natural to measure the difference $m_s - m_{ud}$ via the Wilson or Lagrangian mass difference $d \equiv am_s^{\mathrm{W}} - am_{ud}^{\mathrm{W}} = am_s^{\mathrm{bare}} - am_{ud}^{\mathrm{bare}}$ since it requires only $Z_S(\mu)$.
- It is natural to measure the ratio m_s/m_{ud} via the PCAC quark mass ratio $r \equiv m_s^{\mathrm{PCAC}}/m_{ud}^{\mathrm{PCAC}}$, as it does not require any further renormalization.

Without $O(a)$-improvement only $1/Z_S^{\mathrm{RI}}$ is needed to obtain renormalized masses from

$$am_{ud}^{\mathrm{sub}} = \frac{d}{r-1}\,, \qquad am_s^{\mathrm{sub}} = \frac{rd}{r-1}\,.$$

With tree-level $O(a)$ improvement, renormalized masses take the form

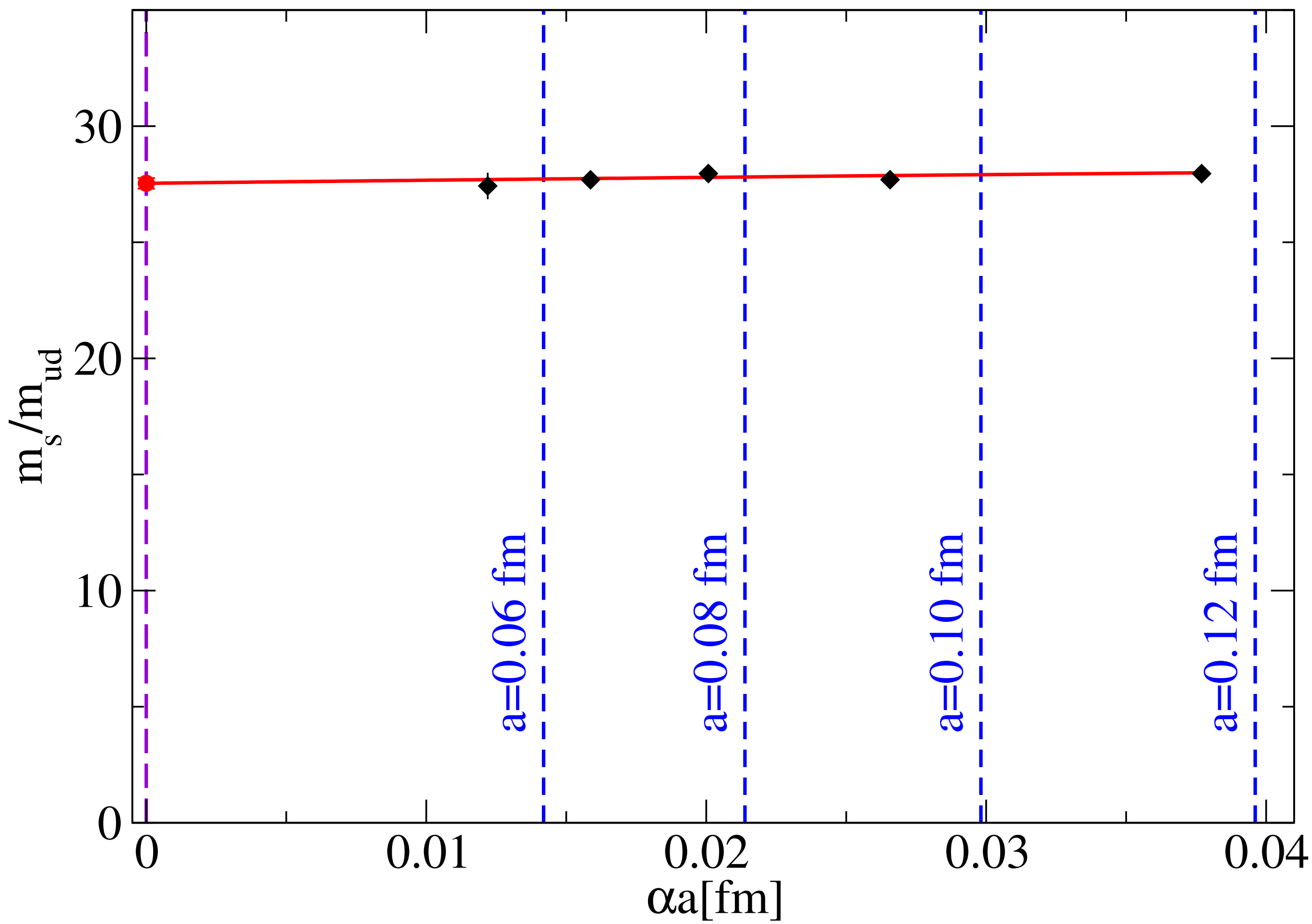
$$m_{ud} = \frac{m_{ud}^{\mathrm{sub}}}{Z_S}\left[1 - \frac{a}{2}(m_{ud}^{\mathrm{sub}} + m_s^{\mathrm{sub}})\right] + O(\alpha a)$$

$$m_s = \frac{m_s^{\mathrm{sub}}}{Z_S}\left[1 - \frac{a}{2}(m_{ud}^{\mathrm{sub}} + m_s^{\mathrm{sub}})\right] + O(\alpha a)\,.$$

$\Longrightarrow$ Advantage 1: only $Z_S^{\mathrm{RI}}(\mu)$ (flavor non-singlet) is required, difficult Z_P not.
$\Longrightarrow$ Advantage 2: no determination of am_{crit} is required.

Final result for m_s/m_{ud}

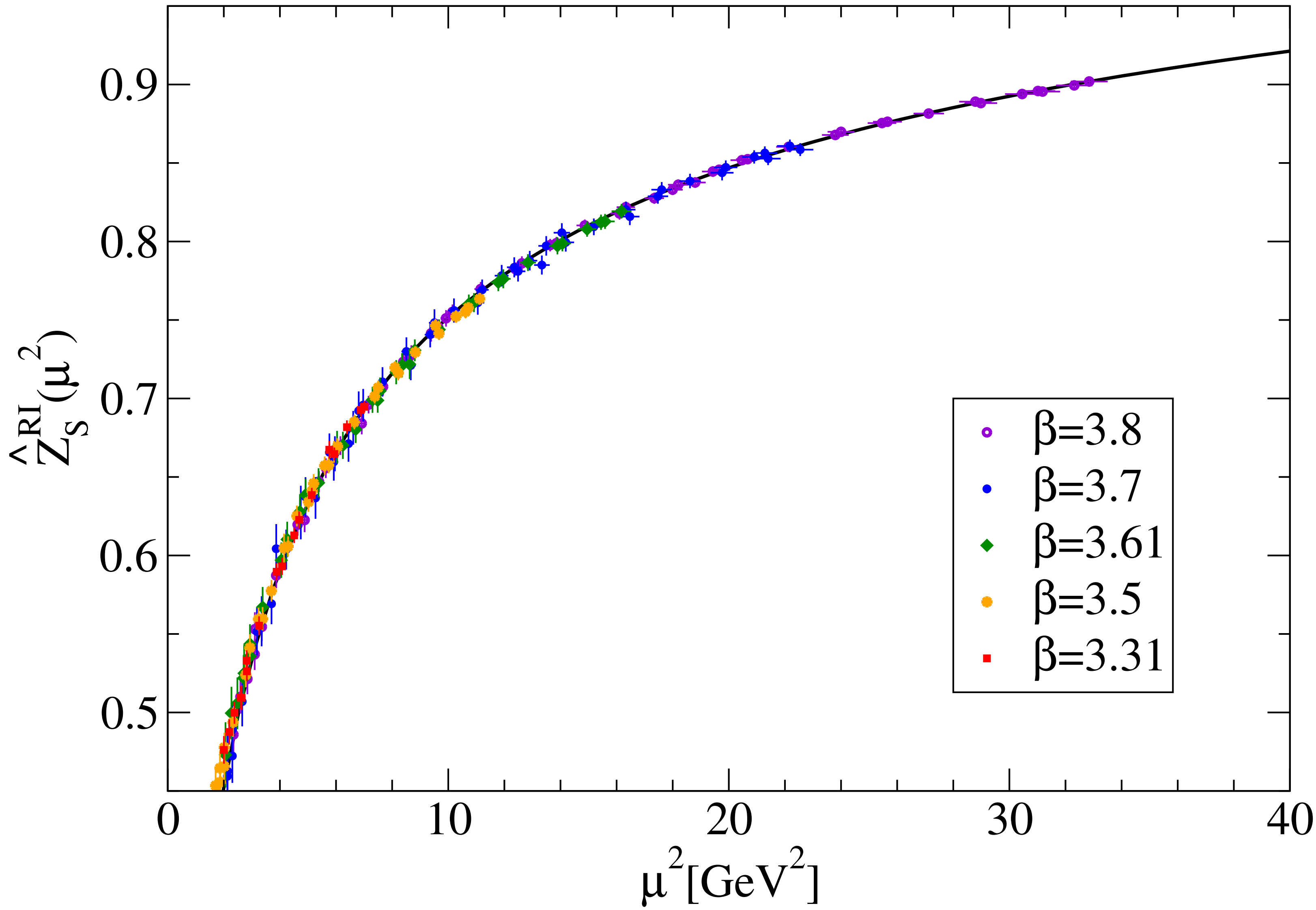
Good scaling of m_s/m_{ud} out to the coarsest lattice ($a\sim 0.116\,\mathrm{fm}$):



Final result is $m_s/m_{ud} = 27.53(20)(08)$ which amounts to 0.78% precision.

2 HEX $N_f = 3$ RI-scheme-running extrapolation for Z_S

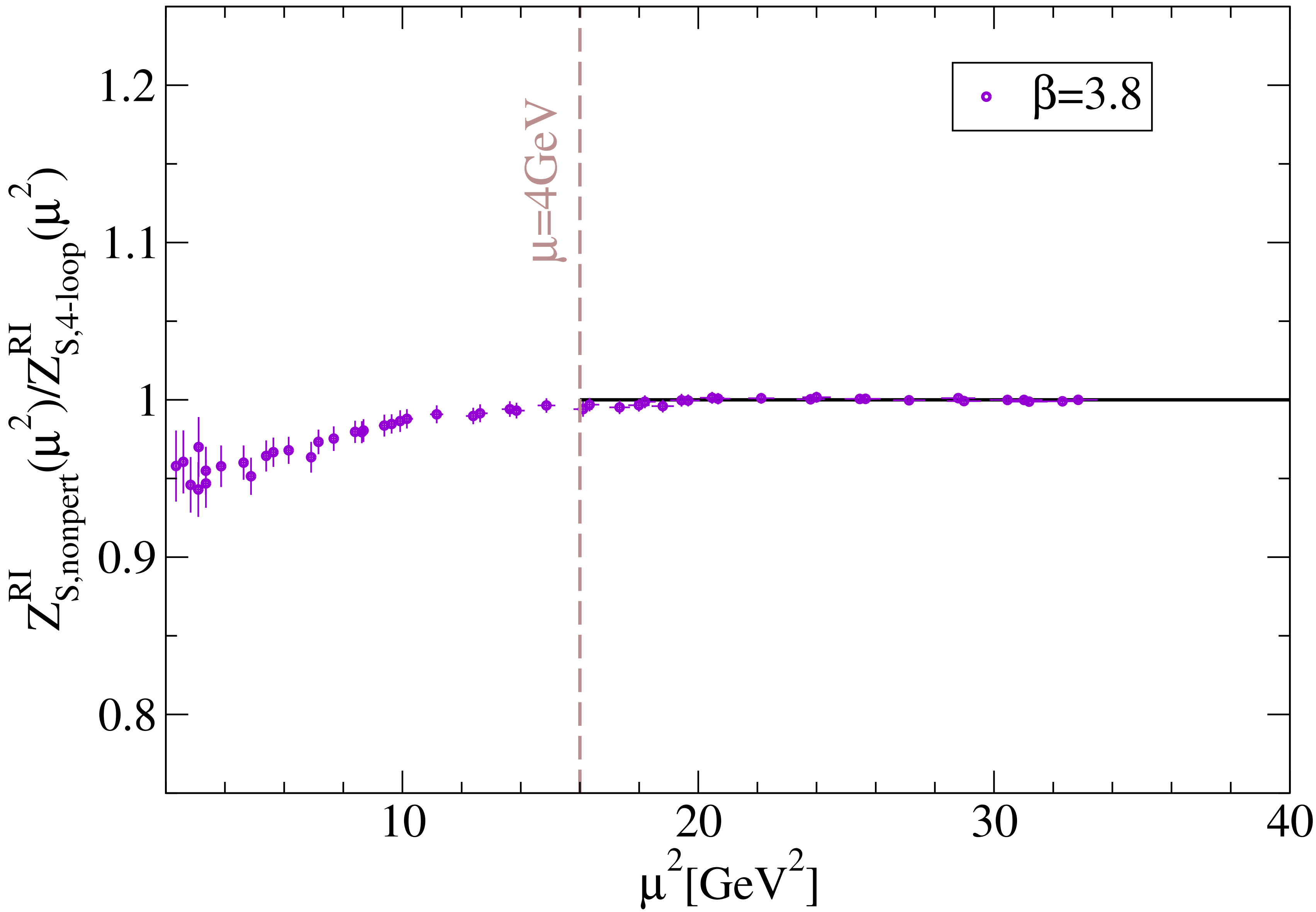
Evolution $Z_S^{\mathrm{RI}}(\mu)/Z_S^{\mathrm{RI}}(4\,\mathrm{GeV})$ has no visible cut-off effects among three finest lattices:



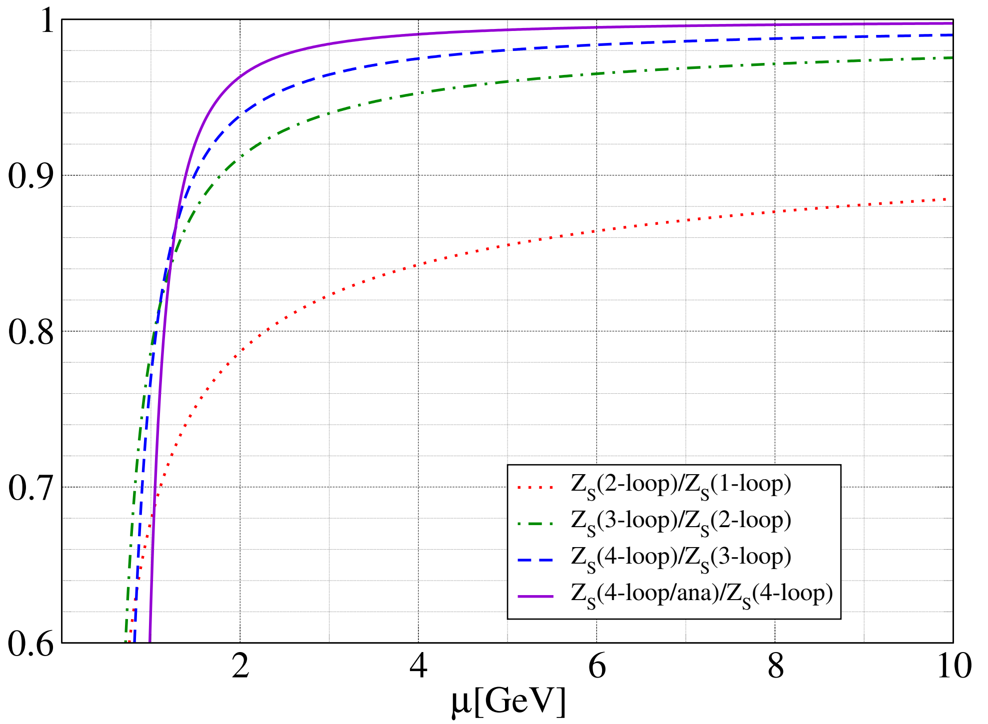
⟶ separate continuum limit with $R_S^{\mathrm{RI}}(\mu, 4\,\mathrm{GeV}) = \lim_{\beta\to\infty} Z_{S,\beta}^{\mathrm{RI}}(4\,\mathrm{GeV})/Z_{S,\beta}^{\mathrm{RI}}(\mu)$

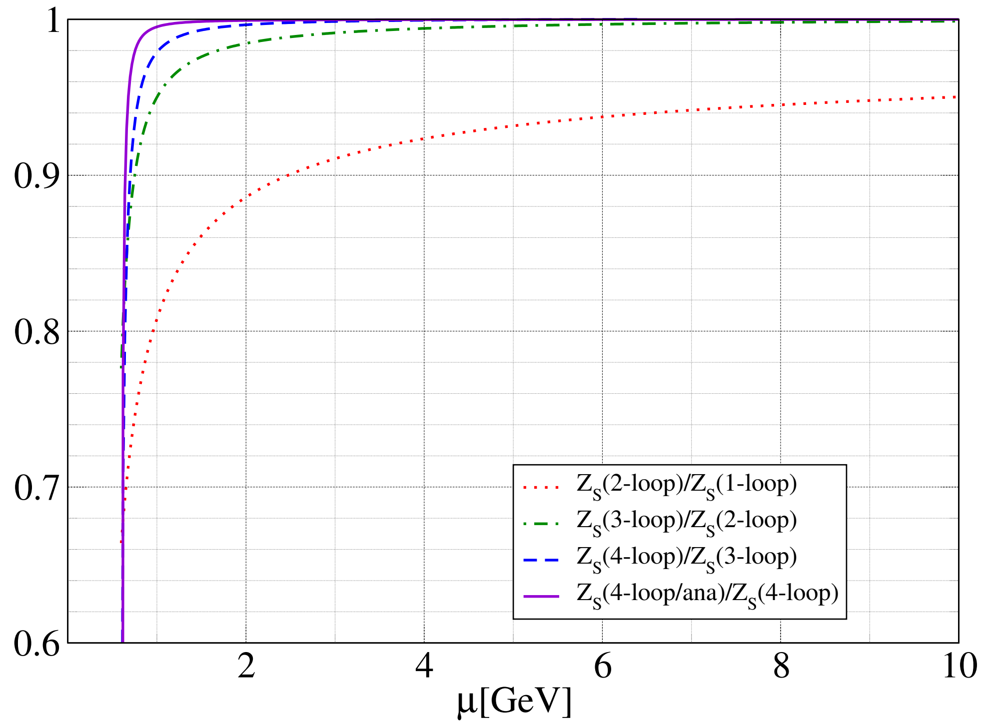
2 HEX $N_f = 3$ RI-scheme-running ratio for Z_S

On the finest lattice we make contact within errors to 4-loop PT for $\mu \geq 4\,\mathrm{GeV}$:



$N_f\!=\!3$ RI- and $\overline{\rm MS}$-scheme perturbative series for Z_S

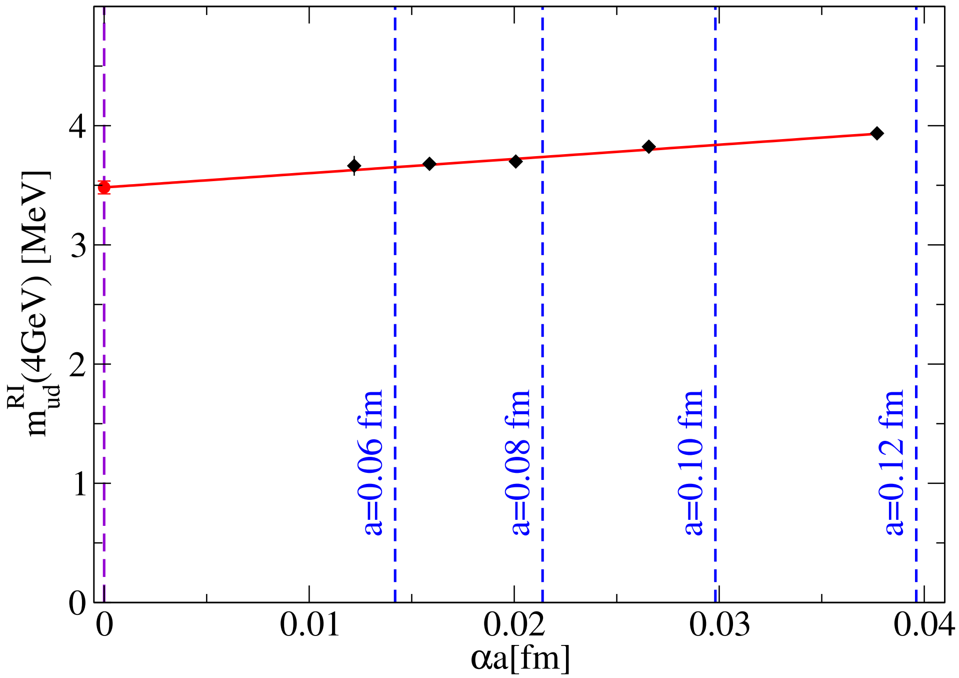


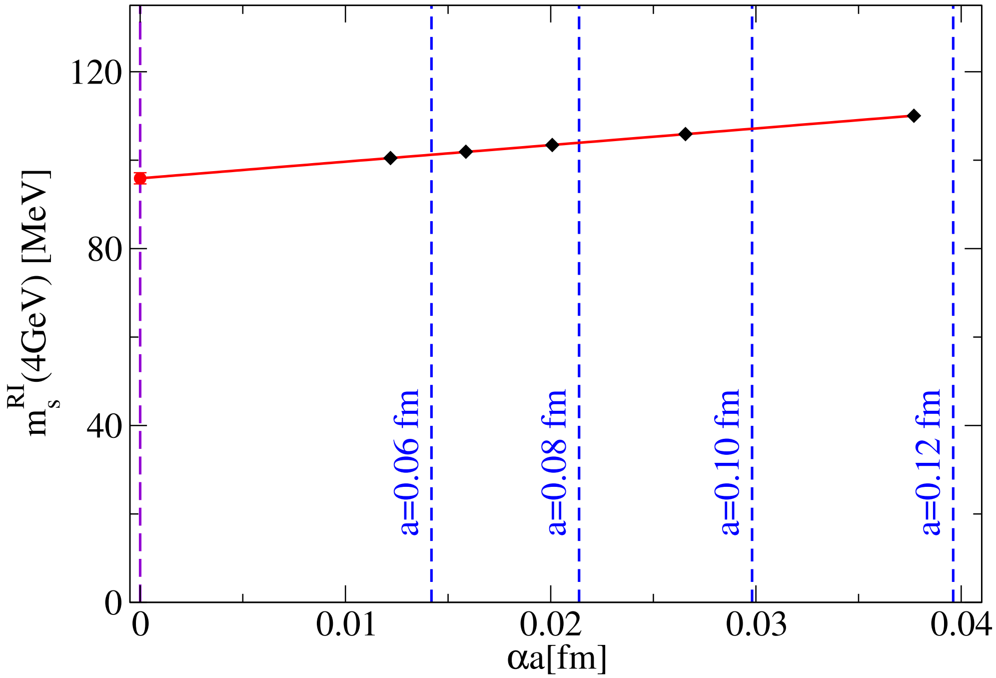


- RI series (left) converges less convincingly than $\overline{\rm MS}$ series (right)
- difference "4-loop" to "4-loop/ana" indicates size of 5-loop effects
- ratio suggests that higher-loop effects in RI are $<\!1\%$ at $\mu\!=\!4\,{\rm GeV}$
- ratio suggests that higher-loop effects in $\overline{\rm MS}$ are negligible down to $\mu\!=\!2\,{\rm GeV}$

Final results for m_s and m_{ud}

Good scaling of $m^{\mathrm{RI}}_{ud,s}(4\,\mathrm{GeV})$ out to the coarsest lattice ($a \sim 0.116\,\mathrm{fm}$):





Conversion with analytical 4-loop formula at $4\,\mathrm{GeV}$ and downwards running in $\overline{\mathrm{MS}}$:

	m_s	m_{ud}	m_u	m_d
RI$(4\,\mathrm{GeV})$	96.4(1.1)(1.5)	3.503(48)(49)	2.17(04)(10)	4.84(07)(12)
RGI	127.3(1.5)(1.9)	4.624(63)(64)	2.86(05)(13)	6.39(09)(15)
$\overline{\mathrm{MS}}(2\,\mathrm{GeV})$	95.5(1.1)(1.5)	3.469(47)(48)	2.15(03)(10)	4.79(07)(12)

Splitting $m_{ud} \to m_u, m_d$ with information from $\eta \to 3\pi$

The process $\eta \to 3\pi$ is highly sensitive to QCD isospin breaking (from $m_u \neq m_d$) but rather insensitive to QED isospin breaking (from $q_u \neq q_d$).

Rewrite the Leutwyler ellipse in the form

$$\frac{1}{Q^2} = 4 \left(\frac{m_{ud}}{m_s}\right)^2 \frac{m_d - m_u}{m_d + m_u}$$

and use the conservative estimate from $\eta \to 3\pi$ given in [Leutwyler CD'09]

$$Q = 22.3(8)$$

together with our result (0.78% precision)

$$m_s/m_{ud} = 27.53(20)(08)$$

to obtain the asymmetry parameter

$$\frac{m_d - m_u}{m_d + m_u} = 0.381(05)(27) \quad \longleftrightarrow \quad m_u/m_d = 0.448(06)(29)$$

from which the results in the last two columns were derived (note: $m_u = 0$ disfavored).

Shortcut via m_{ud}/m_s, m_s/m_c and m_c input from elsewhere

- **HPQCD Collaboration, PRL 104, 132003 (2010) [arXiv:0910.3102]**

Obtain $m_c/m_s = 11.85(16)$ with HISQ quarks on $N_f = 2+1$ asqtad ensembles by MILC. Using their $m_c^{\overline{\rm MS}} = 1.095(11)\,\mathrm{GeV}$ [scale $\mu = 2\,\mathrm{GeV}$ throughout] from an earlier study they obtain $m_s^{\overline{\rm MS}} = 92.4(1.5)\,\mathrm{MeV}$.

- **ETM Collaboration, PRD 82, 114513 (2010) [arXiv:1010.3659]**

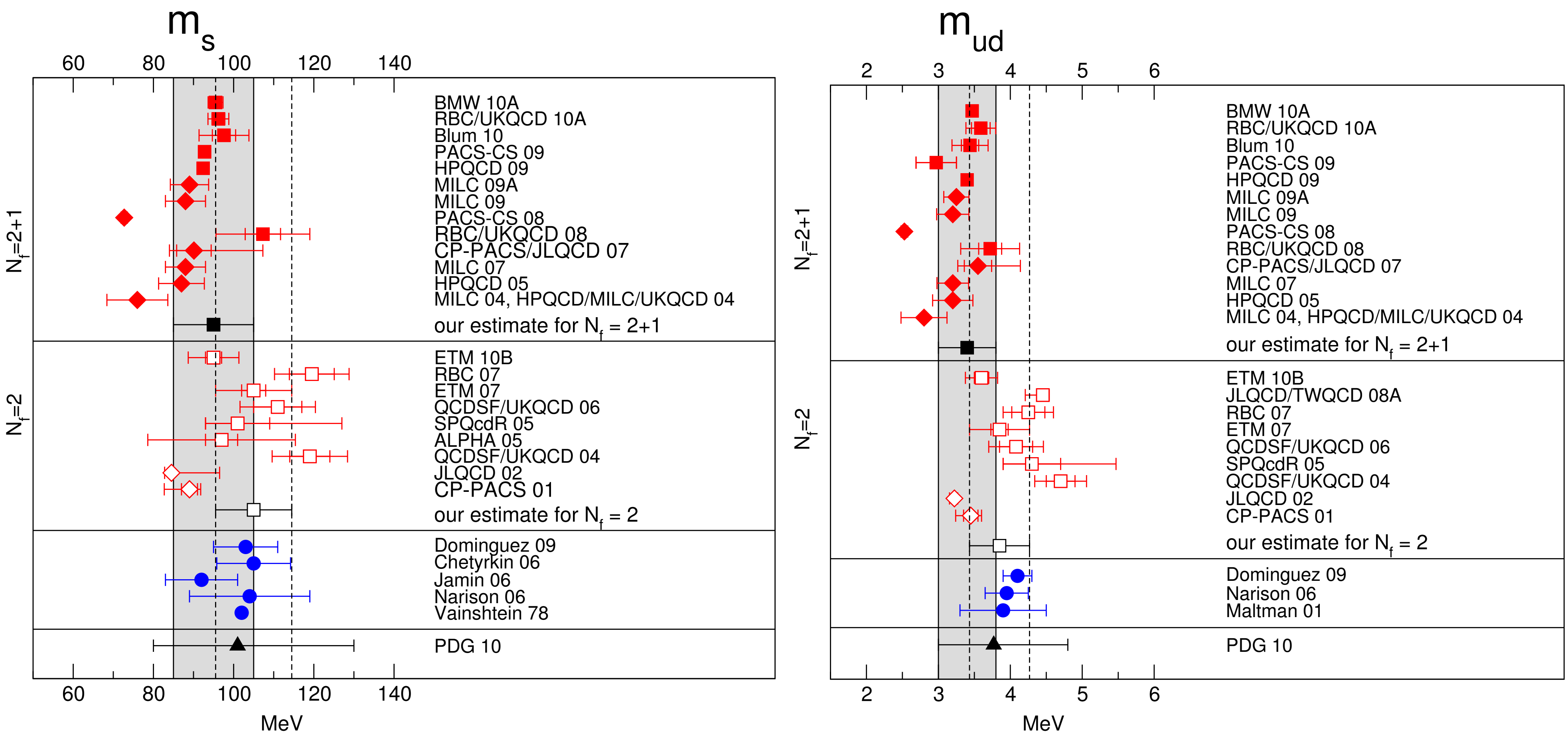
Obtain $m_c/m_s = 12.0(3)$ with Osterwalder-Seiler quarks on their own $N_f = 2$ twisted mass ensembles. They do not use the shortcut, as they prefer to compute m_s with non-perturbative renormalization, finding $m_s^{\overline{\rm MS}} = 95(6)\mathrm{MeV}$.

- **S. Dürr and G. Koutsou, arXiv:1108.1650**

Obtain $m_c/m_s = 11.34(40)(21)$ with Brillouin quarks on $N_f = 2$ clover ensembles by QCDSF. Using an aggregate value $m_c^{\overline{\rm MS}} = 1.093(13)\,\mathrm{GeV}$ from the literature yields $m_s^{\overline{\rm MS}} = 96.4(3.4)(2.1)\,\mathrm{MeV}$.

$\Longrightarrow$ price to pay is that "anchor" m_c typically includes a perturbative uncertainty

FLAG compilation



$\longrightarrow$ apparent "tension" between $N_f = 2$ (white band) and $N_f = 2{+}1$ (grey band) results may be due to better NP renormalization in the latter case.

$\longrightarrow$ FLAG estimates are *significantly more precise* than PDG estimates.

$\longrightarrow$ BMW collaboration values not yet included $\Longrightarrow$ arXiv:1011.2403,1011.2711.

Outlook: $N_f = 1{+}1{+}1{+}1$ simulations with electromagnetism

- 2002-20??:

  $N_f = 2{+}1$ QCD requires 3 polished input values (e.g. M_π, M_K, M_Ω in theory with $m_u, m_d \to (m_u{+}m_d)/2$ and $e \to 0$)

  $\longrightarrow$ analysis suggests $M_\pi{=}134.8(3)\mathrm{MeV}, M_K{=}494.2(5)\mathrm{MeV}$ (see FLAG report)

- 2010-????:

  $N_f = 2{+}1{+}1$ QCD requires 4 polished input values (like above plus M_{D_s}, still $m_u, m_d \to (m_u{+}m_d)/2$ and $e \to 0$)

  $\longrightarrow$ charm unquenched, but no conceptual change on isospin issue

- 2014-????:

  $N_f = 1{+}1{+}1{+}1$ QCD requires 5 input variables (e.g. $M_{\pi^\pm}, M_{K^\pm}, M_{K^0}, M_{D_s}, M_\Omega$)

  $\longrightarrow$ requires disconnected contribution to flavor-singlet quantities

  $\longrightarrow$ analysis of π^0-η-γ mixing mandatory to extract physical masses

  $\longrightarrow$ QED and QCD renormalization intertwined (m_s/m_d is RGI, m_u/m_d is not)

  $\longrightarrow$ final word on $m_u \overset{?}{=} 0$ [in QCD+QED] will be possible