f_K/f_π in full QCD

Stephan Dürr

von Neumann Institute for Computing
FZ Jülich and DESY Zeuthen

in collaboration with

Z. Fodor, C. Hoelbling, S.D. Katz, S. Krieg, L. Lellouch,
T. Kurth, T. Lippert, K.K. Szabo, G. Vulvert

[Budapest-Marseille-Wuppertal Collaboration]

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Introduction: why f_K/f_π ?

W.J. Marciano, PRL 93 231803 (2004) [hep-ph/0402299]:

- $|V_{\rm ud}|$ is known, from super-allowed nuclear β-decays, with 0.03% precision.

- $|V_{\rm us}|$ is much less precisely known, but can be linked to $|V_{\rm ud}|$ via a relation involving f_K/f_π, with everything else known rather accurately:

$$\frac{\Gamma(K\to l\bar\nu_l)}{\Gamma(\pi\to l\bar\nu_l)}=\frac{|V_{\rm us}|^2}{|V_{\rm ud}|^2}\frac{f_K^2}{f_\pi^2}\frac{M_K(1-m_l^2/M_K^2)^2}{M_\pi(1-m_l^2/M_\pi^2)^2}\{1+\frac{\alpha}{\pi}(C_K-C_\pi)\}$$

- CKM unitarity $|V_{\rm ud}|^2+|V_{\rm us}|^2+|V_{\rm ub}|^2=1$ (with $|V_{\rm ub}|$ being negligibly small) is genuine to the SM; any deviation is a *model-independent* signal of BSM physics.

$\Longrightarrow$ calculate f_K/f_π in $N_f=2{+}1$ QCD (with quark masses extrapolated to the physical point) on the lattice; the precision attained gives the precision of $|V_{\rm us}|$.

Setup: dominant sources of uncertainty

What we do to control systematic uncertainties ("full QCD shopping list"):

(a) $N_f = 2{+}1$ with exact algorithm (and universality class of QCD)

(b) complete baryon octet/decuplet spectrum to set the scale

(c) large spatial volumes $(M_\pi L \geq 4)$ to assure small finite volume effects

(d) sufficiently chiral data $(M_\pi \simeq 190\,\mathrm{MeV})$ for small extrapolation range

(e) no less than 3 lattice spacings to assure controlled continuum extrapolation

Setup: action, algorithm, resources

- action:

  tree-level $O(a^2)$ improved Symanzik glue and tree-level $O(a)$ improved fat-clover quarks [6 levels of $\alpha = 0.11$ stout smearing, both in covariant derivative and in $F_{\mu\nu}$]

- algorithm:

  HMC/RHMC with even-odd preconditioning, multiple time-scale Omelyan integration, Hasenbusch acceleration and mixed precision solver [Clark et al '06, Sexton Weingarten '92, Omelyan et al '03, Hasenbusch '01, Urbach et al '06, BMW '08]

- resources:

  BG/L @ FZJ: 2005-2008, $8 * 2048 = 16384$ processors PowerPC440 @ 700MHz (2.8 GFlops each), 1GB per node, 3D torus network, 46/37 Tflops peak/sustained

  BG/P @ FZJ: since 2008, $16 \cdot 4096 = 65536$ processors PowerPC450 @ 850MHz (3.4 GFlops each), 2GB per node, 3D torus network, 223/180 Tflops peak/sustained

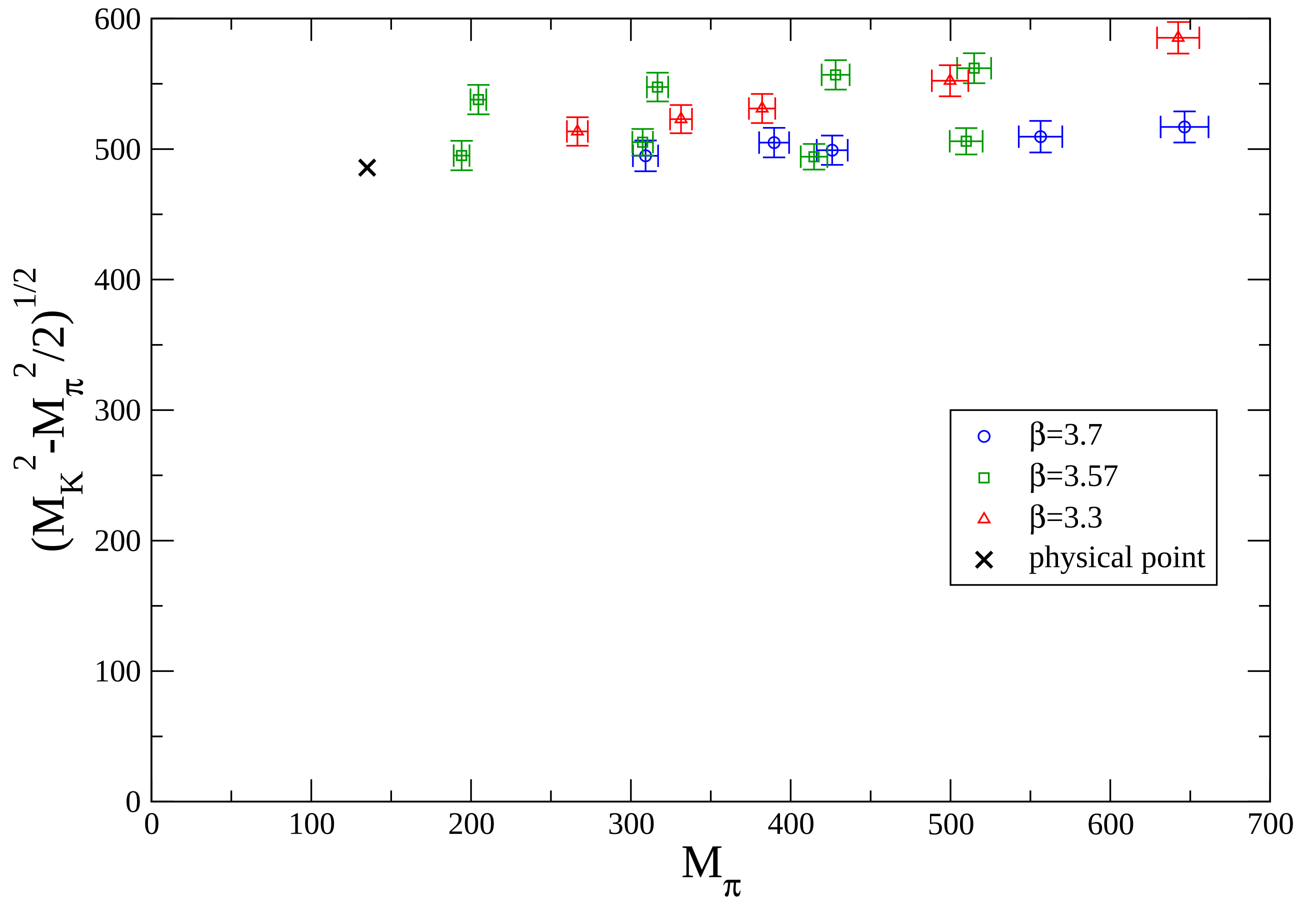
Setup: setting m_{ud}, m_s and a^{-1}

We set m_{ud}, m_s, a^{-1} through M_π, M_K, M_Ξ (or M_Ω).

$\longrightarrow$ S. Krieg: *The hadron spectrum in full QCD: setup and parameter selection*

$\longrightarrow$ Ch. Hoelbling: *The hadron spectrum in full QCD: analysis details and final result*

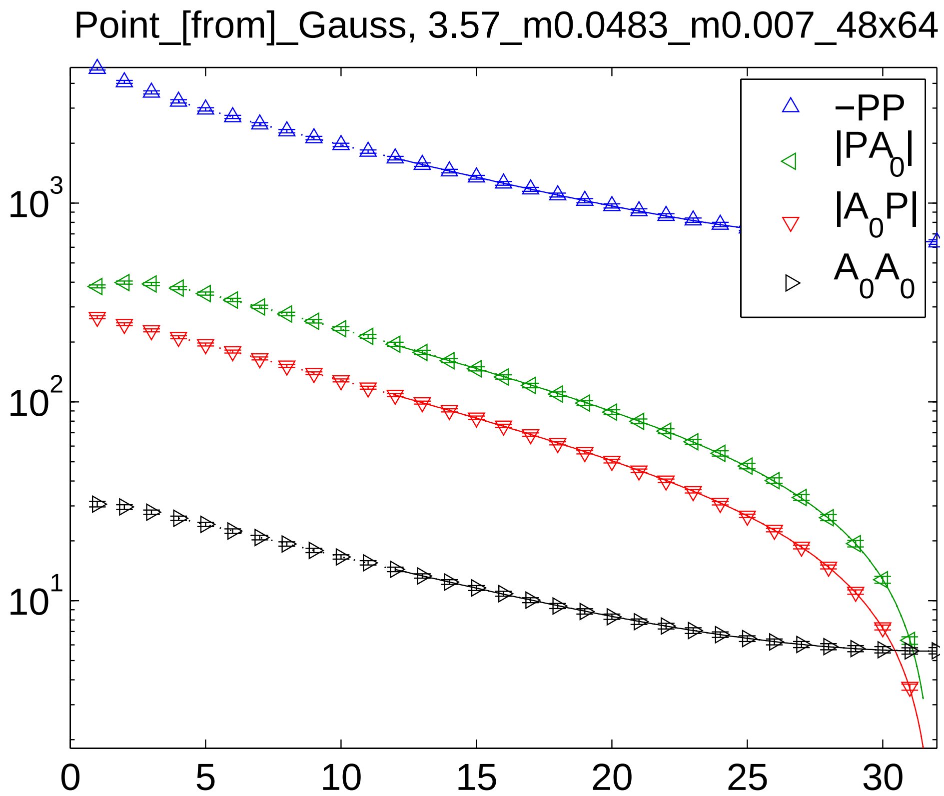
We fix bare strange mass such that renormalized m_s is correct at physical m_{ud} point:

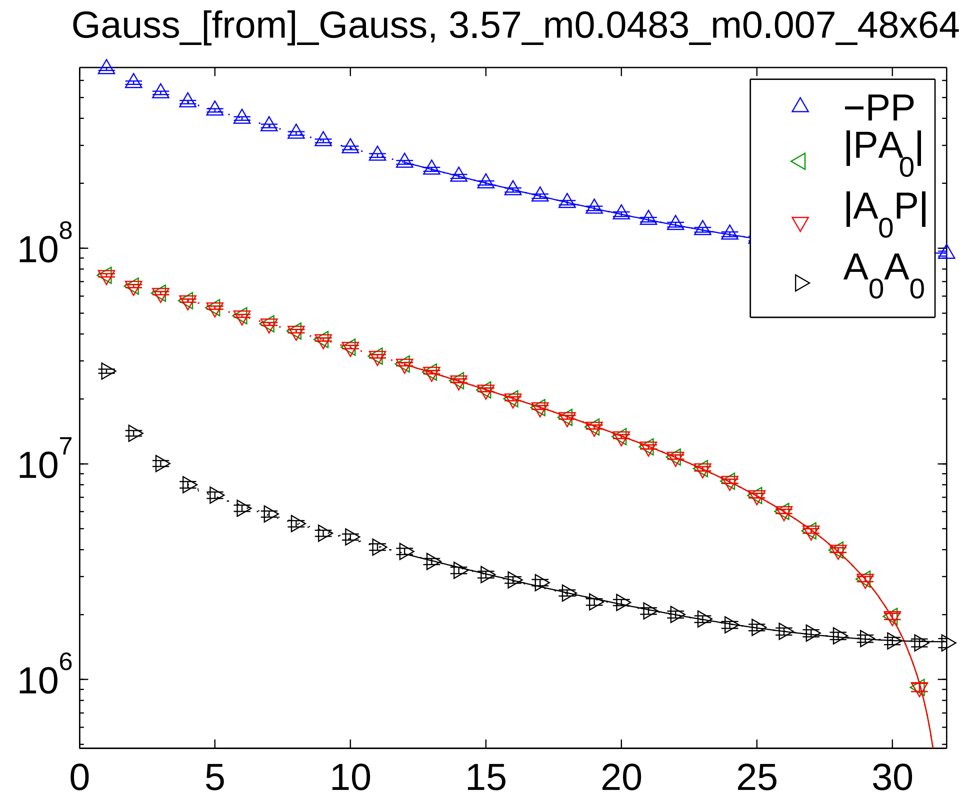


$\Longrightarrow$ extract f_K/f_π from "unitary" data and extrapolate to the physical mass point !

Analysis: combined fits

Excellent data quality even on our lightest ensemble ($M_\pi \simeq 190\,\mathrm{MeV}$ and $L \simeq 4.0\,\mathrm{fm}$):





$\cosh(.)/\sinh(.)$ for $-PP, |PA_0|, |A_0P|, A_0A_0$ with Gauss source and local/Gauss sink

$C_{Xx,Yy}(t) = c_0 e^{-M_0 t} \pm c_0 e^{-M_0(T-t)} + \ldots$ with $X, Y \in \{P, A_0\}$ and $x, y \in \{\mathrm{loc}, \mathrm{gau}\}$

$\longrightarrow$ $c_0 = G\tilde{G}/M_0, G\tilde{F}, F\tilde{G}, F\tilde{F}M_0$ (left) and $c_0 = \tilde{G}\tilde{G}/M_0, \tilde{G}\tilde{F}, \tilde{F}\tilde{G}, \tilde{F}\tilde{F}M_0$ (right)

$\longrightarrow$ combined 1-state fit of 8 correlators with 5 parameters yields $M_\pi, F_\pi, m_{\mathrm{PCAC}}$

Analysis: chiral extrapolation

- Chiral $SU(3)$ formula:

$$\frac{F_K}{F_\pi} = 1+\frac{1}{32\pi^2F_0^2}\Big\{\frac{5}{4}M_\pi^2\log(\frac{M_\pi^2}{\mu^2})-\frac{1}{2}M_K^2\log(\frac{M_K^2}{\mu^2}) -[M_K^2-\frac{1}{4}M_\pi^2]\log(\frac{4M_K^2-M_\pi^2}{3\mu^2})\Big\}+\frac{4}{F_0^2}[M_K^2-M_\pi^2]L_5$$

- Chiral $SU(2)$_plus_strange formula [arXiv:0804.0473 by RBC/UKQCD, simplified]:

$$\frac{F_K}{F_\pi}=\frac{F_K}{F_\pi}\Big|_{m_{ud}=0}\left\{1+\frac{5}{8}\frac{M_\pi^2}{(4\pi F)^2}\log\Big(\frac{M_\pi^2}{\Lambda^2}\Big)\right\}$$

- Polynomial expansion $F_\pi/F_K=d_0+d_1M_\pi+d_2M_\pi^2$ (e.g. around $300\,\mathrm{MeV}$) at fixed physical m_s, together with constraint $F_K=F_\pi$ at $M_K=M_\pi$, suggests:

$$\frac{F_K}{F_\pi}=\frac{c_0+c_1M_K+c_2M_K^2}{c_0+c_1M_\pi+c_2M_\pi^2}$$

⟶ Use all of them and treat spread as indicative of systematic uncertainty !

Analysis: continuum extrapolation

Q: Should we use dedicated version of Chiral Perturbation Theory (XPT) ?

A: Aoki Bär Takeda Ishikawa PRD73, 014511 (2006) for clover fermions:

$$
\begin{aligned}
M_\pi^2 &= B_0 2m\Big\{1+\mu_\pi-\frac{1}{3}\mu_\eta+2mK_3+K_4-\frac{H''}{F_0^2}\Big\}\\
M_K^2 &= B_0(m+m_s)\Big\{1+\frac{2}{3}\mu_\eta+(m+m_s)K_3+K_4-\frac{H''}{F_0^2}\Big\}\\
F_\pi &= F_0\Big\{1-2\mu_\pi-\mu_K+2mK_6+K_7-\frac{H'}{F_0^2}\Big\}\\
F_K &= F_0\Big\{1-\frac{3}{4}\mu_\pi-\frac{3}{2}\mu_K-\frac{3}{4}\mu_\eta+(m+m_s)K_6+K_7-\frac{H'}{F_0^2}\Big\}
\end{aligned}
$$

$\longrightarrow$ Redefine $B_0\{1-H''/F_0^2\}\to B_0'$ and terms with H'' will disappear

$\longrightarrow$ Redefine $F_0\,\{1-H'/F_0^2\}\to F_0'$ and terms with H' will disappear

Conjecture: dedicated WXPT calculations for M_P, F_P not needed; with $m\equiv m_{\rm PCAC}$ effects of finite lattice spacing are taken into account via augmenting all low-energy constants by factors of $(1+\mathrm{const}\,a^2)$ to any chiral order. (not needed below)

Analysis: infinite volume extrapolation

- Finite volume effects on F_K, F_π are known at the 2-loop level [Colangelo et al. '05]

$$\frac{F_\pi(L)}{F_\pi} = 1+\sum_{n=1}^{\infty}\frac{m(n)}{\sqrt{n}}\frac{1}{M_\pi L}\;1\;\frac{M_\pi^2}{(4\pi F_\pi)^2}\left[I_{F_\pi}^{(2)}+\frac{M_\pi^2}{(4\pi F_\pi)^2}I_{F_\pi}^{(4)}+\ldots\right]$$

$$\frac{F_K(L)}{F_K} = 1+\sum_{n=1}^{\infty}\frac{m(n)}{\sqrt{n}}\frac{1}{M_\pi L}\frac{F_\pi}{F_K}\frac{M_\pi^2}{(4\pi F_\pi)^2}\left[I_{F_K}^{(2)}+\frac{M_K^2}{(4\pi F_\pi)^2}I_{F_K}^{(4)}+\ldots\right]$$

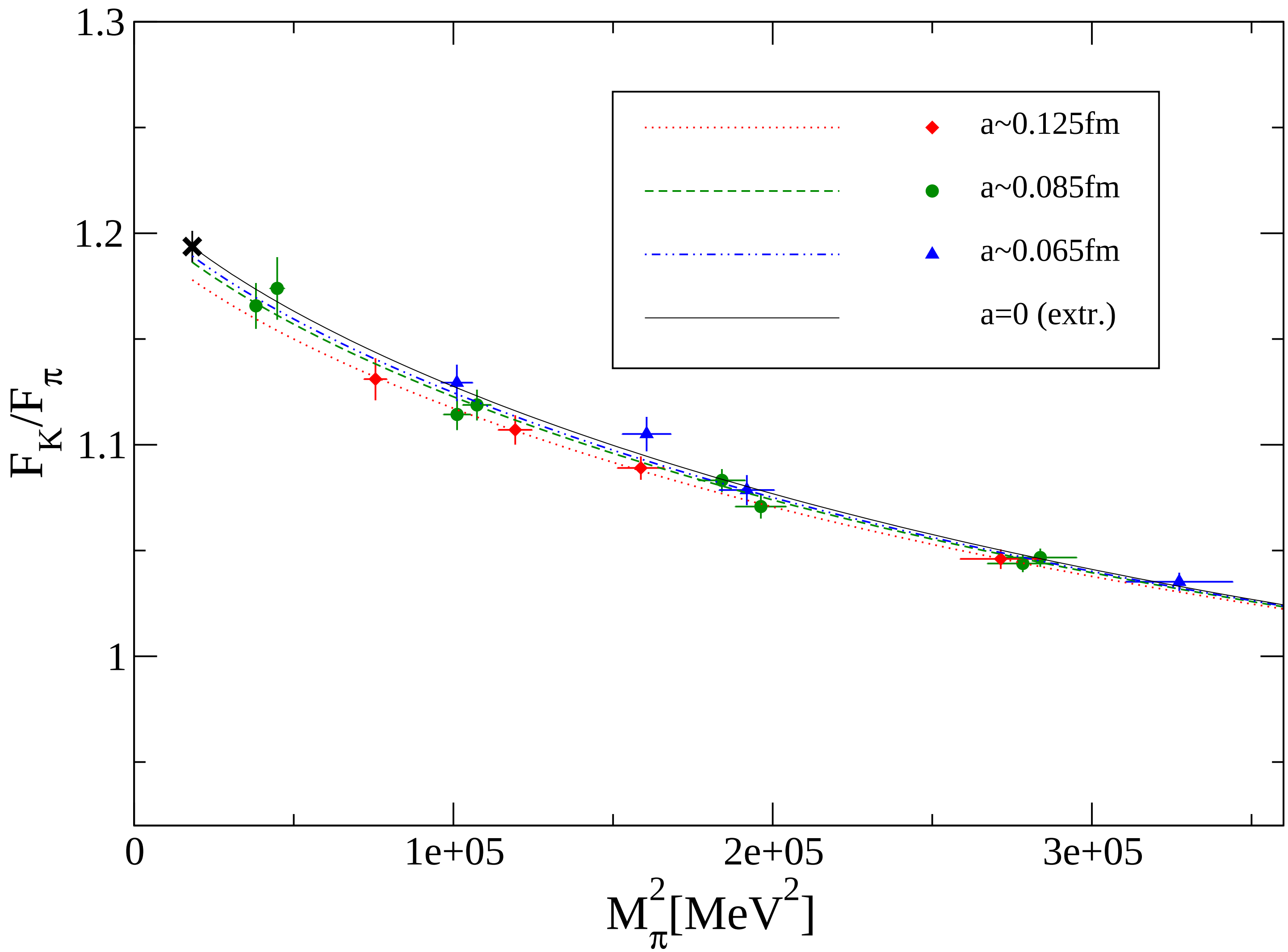
with $I_{F_\pi}^{(2)}=-4K_1(\sqrt{n}\,M_\pi L)$ and $I_{F_K}^{(2)}=-\frac{3}{2}K_1(\sqrt{n}\,M_\pi L)$, where $K_1(.)$ is a Bessel function of the second kind, and lengthy expressions for $I_{F_\pi}^{(4)}, I_{F_K}^{(4)}$.

- In the ratio finite volume effects cancel partly; evident from the 1-loop formula

$$\frac{F_K(L)}{F_\pi(L)} = \frac{F_K}{F_\pi}\left\{1+\sum_{n=1}^{\infty}\frac{m(n)}{\sqrt{n}}\frac{1}{M_\pi L}\frac{M_\pi^2}{(4\pi F_\pi)^2}\left[\frac{F_\pi}{F_K}I_{F_K}^{(2)}-I_{F_\pi}^{(2)}\right]\right\}\;.$$

- We calculate $\frac{F_K(L)}{F_\pi(L)}/\frac{F_K}{F_\pi}$ at 1-loop and 2-loop level, and $F_\pi(L)/F_\pi$ at 2-loop level.

Result: combined fits

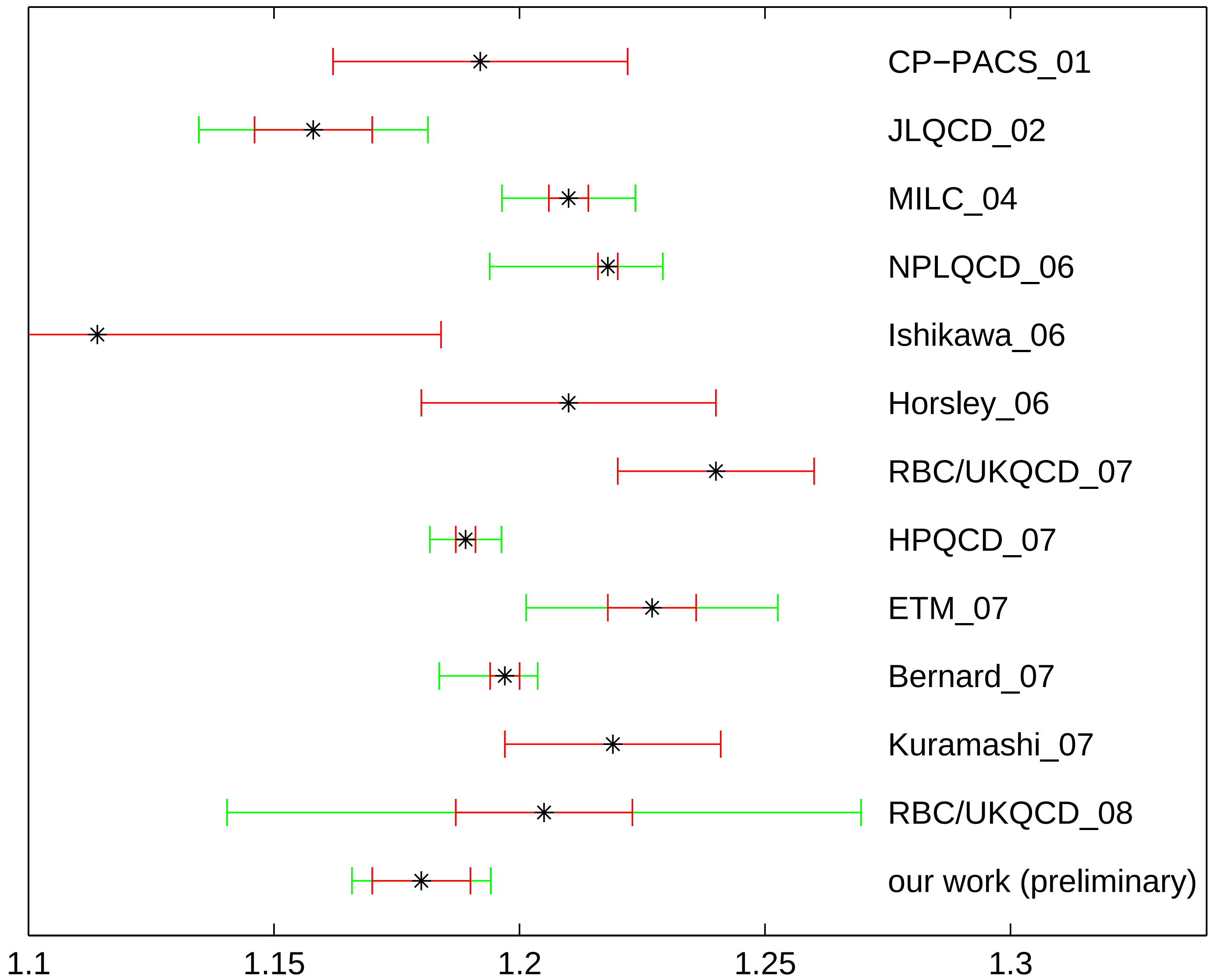


$\longrightarrow$ plot shows $\mathrm{data}(M_\pi^2, 2M_K^2 - M_\pi^2) - \mathrm{fit}(M_\pi^2, 2M_K^2 - M_\pi^2) + \mathrm{fit}(M_\pi^2, [2M_K^2 - M_\pi^2]_{\mathrm{phys}})$.

$\longrightarrow$ f_K/f_π scales rather nicely [we have $a^2/\mathrm{fm}^2 = 0.0042, 0.0072, 0.0156$].

$\Longrightarrow$ $f_K/f_\pi = 1.18(1)(1)$ at the physical m_{ud}, in the continuum, for infinite volume.

Result: errorbars over time



Finish: update on $|V_{\rm us}|$

- Average $|V_{\rm ud}| = 0.97377(27)$ [PDG'06] and $0.97418(26)$ [Towner'07] to give

$$|V_{\rm ud}| = 0.97398(18)(20) = 0.97398(27)\;.$$

- Plug experimental information $\Gamma(K \to \mu\bar{\nu})/\Gamma(\pi \to \mu\bar{\nu}) = 1.3337(39)$ [PDG'06] and $C_K - C_\pi = -3.0 \pm 1.5$ [Marciano] into Marciano's equation (first slide); this yields

$$\frac{|V_{\rm us}|}{|V_{\rm ud}|}\,\frac{f_K}{f_\pi} = 0.2757(6)\;.$$

- Upon combining the previous one/two points and our value for f_K/f_π we obtain

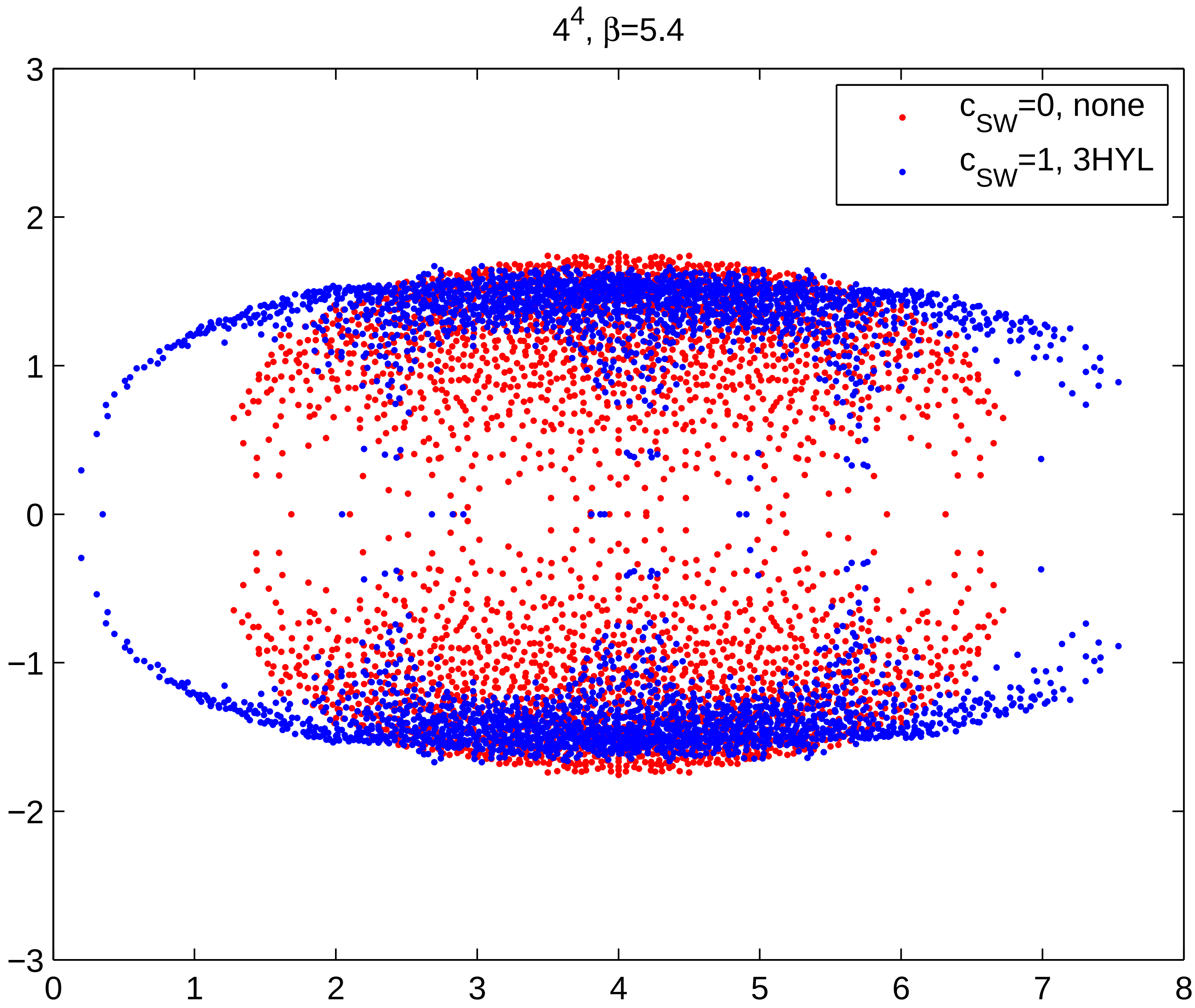
$$|V_{\rm us}|/|V_{\rm ud}| = 0.2336(28) \quad \text{and} \quad |V_{\rm us}| = 0.2276(27)\;.$$

- Upon including $|V_{\rm ub}| = (4.31 \pm 0.30)\,10^{-3}$ [PDG'06] we end up with

$$|V_{\rm ud}|^2 + |V_{\rm us}|^2 + |V_{\rm ub}|^2 = 1.0004(14)\;.$$

BACKUP SLIDES

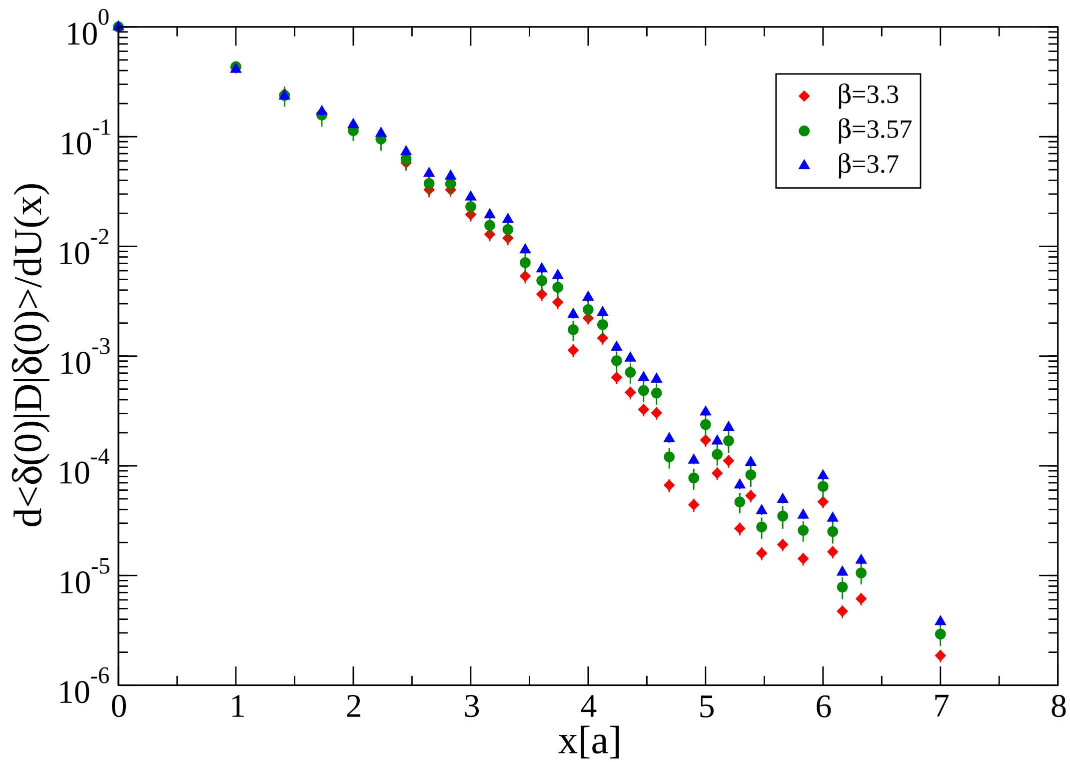
Backup: why smear?



Backup: action locality

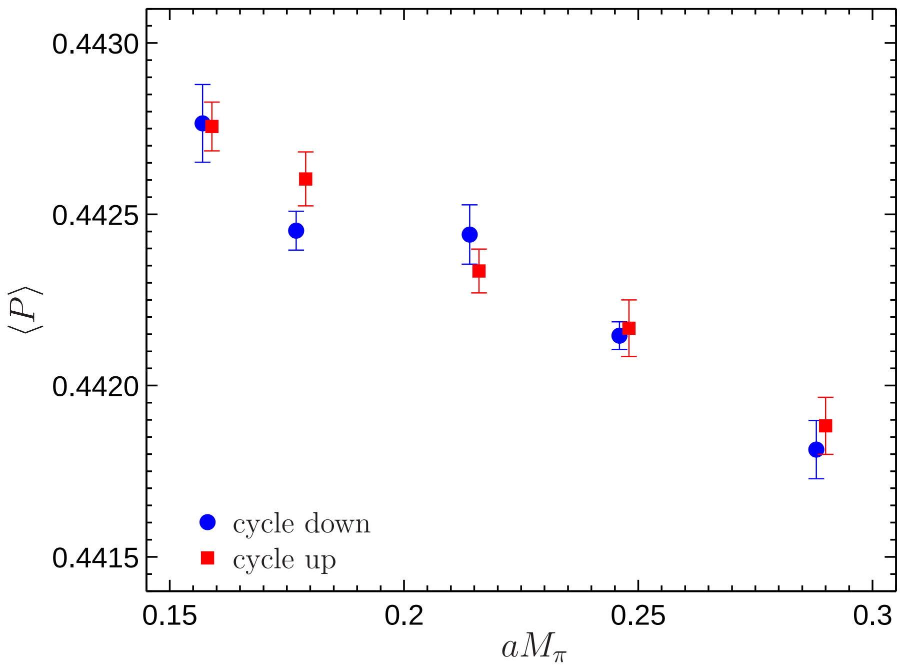
- locality in position space:
  $|D(x,y)| < \mathrm{const}\, e^{-\lambda|x-y|}$ with $\lambda = O(a^{-1})$ for all couplings.
  Our case: $D(x,y) = 0$ as soon as $|x-y| > 1$ (despite 6 smearings).

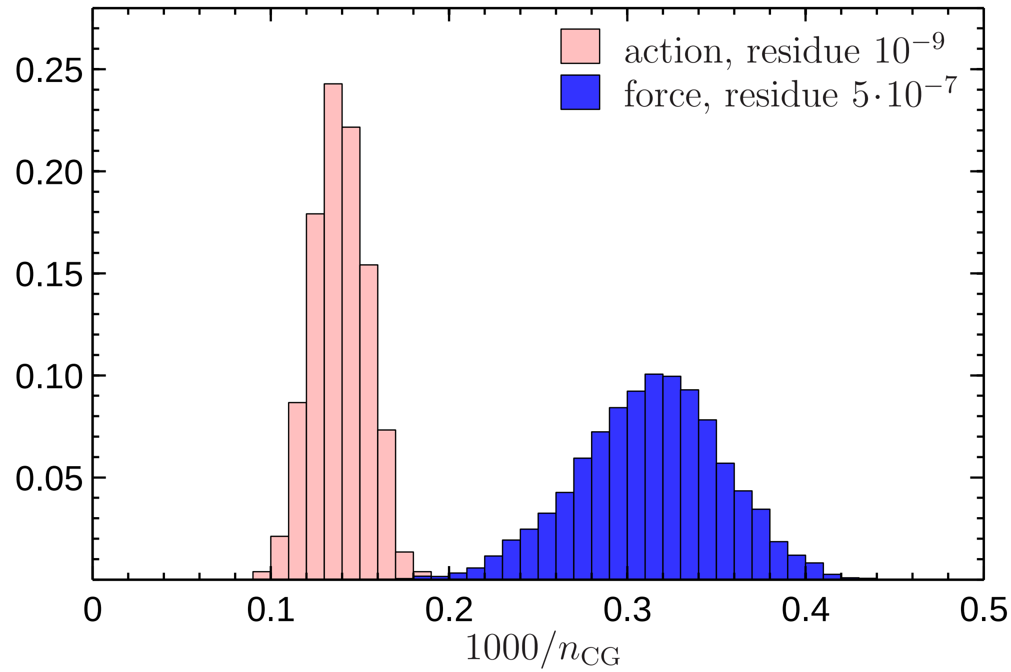
- locality in space of gauge fields:
  $|\delta D(x,y)/\delta A(z)| < \mathrm{const}\, e^{-\lambda|(x+y)/2-z|}$ with $\lambda = O(a^{-1})$ for all couplings.
  Our case: $\delta D(x,x)/\delta A(z) < \mathrm{const}\, e^{-\lambda|x-z|}$ with $\lambda \simeq 2.2a^{-1}$ for $2 \le |x-z| \le 6$.



Backup: algorithmic stability

No hysteresis effects and 5σ-stability in light quark mass production runs:





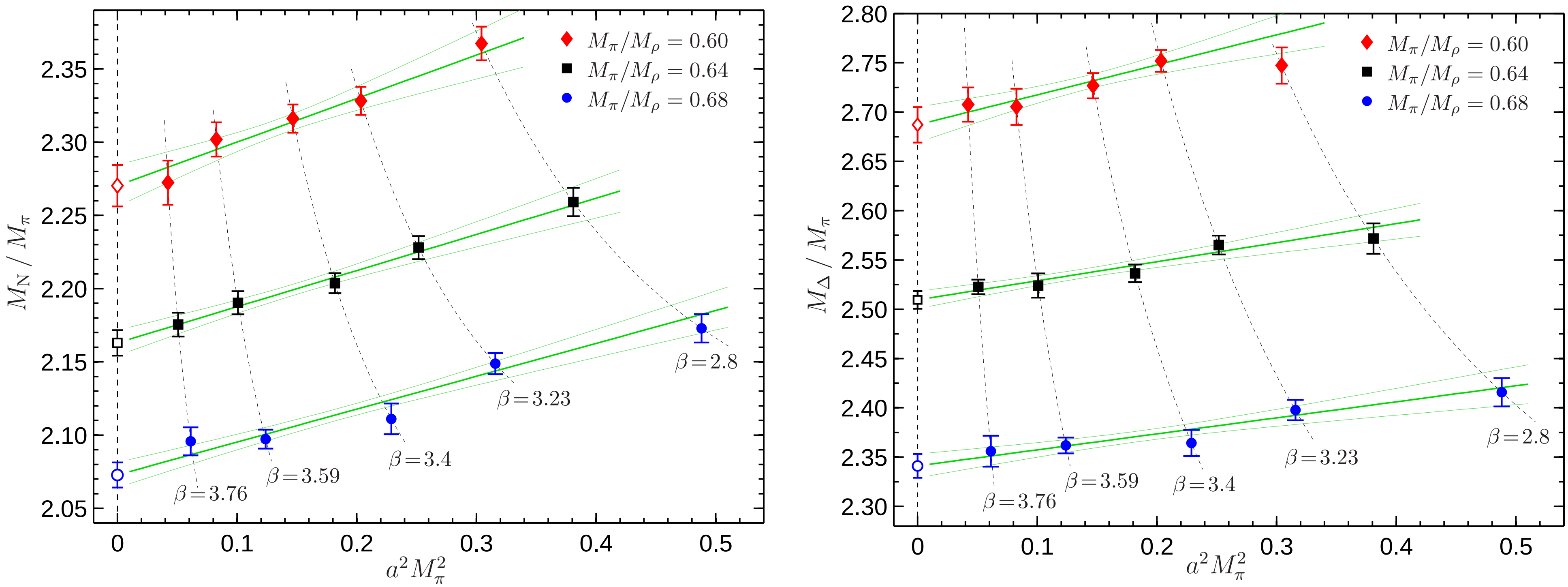
$\longrightarrow$ to monitor $1/N_{\rm CG}$ is cheaper than minimal eigenvalue of $|\gamma_5 D|$

$\longrightarrow$ also $R_{\rm acc}$ and $e^{-\Delta H}$ are being monitored

$\longrightarrow$ see arxiv:0802.2706 [BMW Collab.] for details

Backup: scaling properties

Explicit scaling test for M_N and M_Δ in $N_f = 3$ QCD:

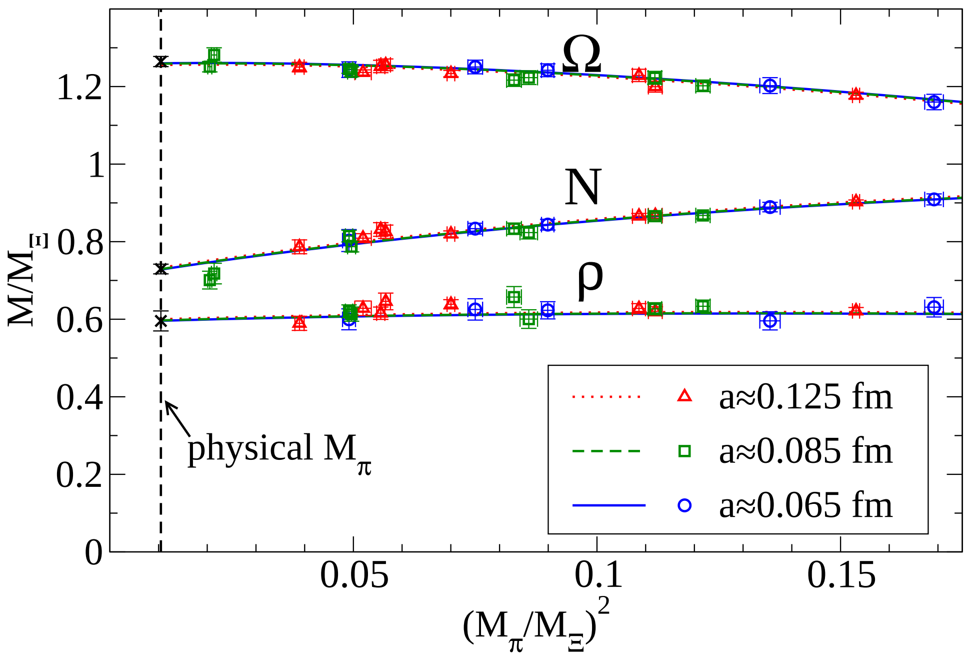


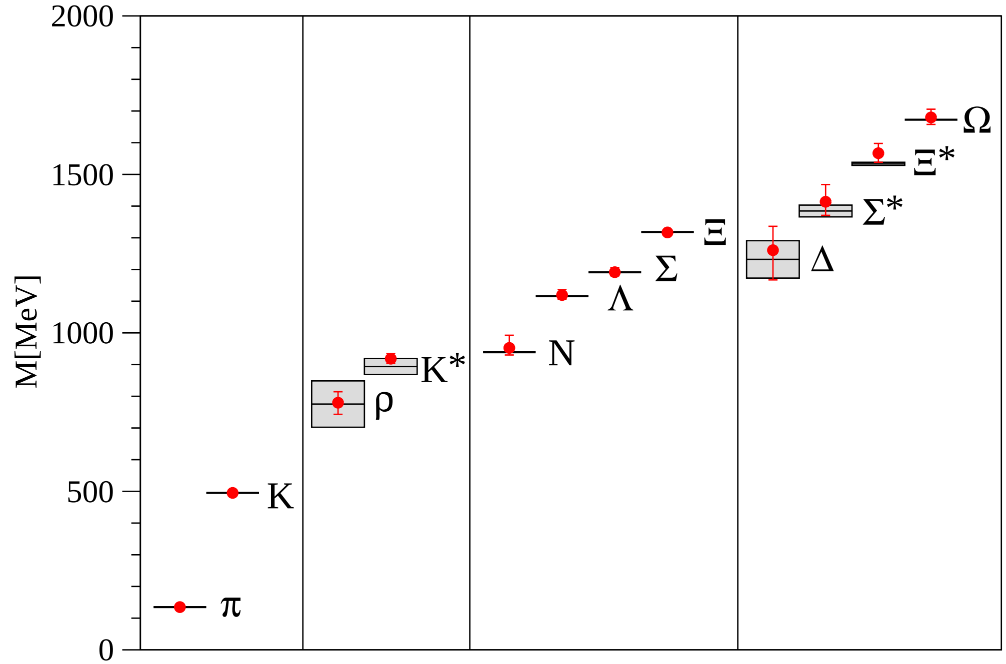
⟶ clean $O(a^2)$ scaling of 6-stout action out to $a \sim 0.15\,\mathrm{fm}$

⟶ see arxiv:0802.2706 [BMW Collab.] for details

⟶ Th. Kurth: *Scaling study of dynamically smeared fermions*

Backup: octet/decuplet spectrum with extrapolation





- large volumes ($M_\pi L \geq 4$ maintained, larger/smaller volumes for check)
- light pions ($M_\pi \sim 190\,\mathrm{MeV}$ at two lattice spacings)
- three couplings ($a \sim 0.065, 0.085, 0.125\,\mathrm{fm}$)

⟶ S. Krieg: *The hadron spectrum in full QCD: setup and parameter selection*
⟶ Ch. Hoelbling: *The hadron spectrum in full QCD: analysis details and final result*